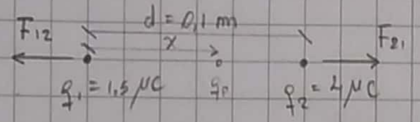
Ley de Coulamb:

1) 

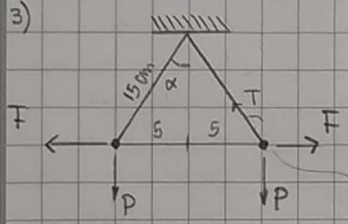
a) $F_{12} = F_{21} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 \cdot q_2}{d^2} = 5{,}39 \cdot 10^{12}\ N$

b) $d_{10} = x \qquad d_{20} = d - x$

$F_{01} = F_{02} \Rightarrow \dfrac{1}{\cancel{4\pi\varepsilon_0}} \dfrac{q_1 \cancel{q_0}}{x^2} = \dfrac{1}{\cancel{4\pi\varepsilon_0}} \dfrac{q_2 \cancel{q_0}}{(d-x)^2} \rightsquigarrow \boxed{x = 3{,}8 \cdot 10^{-2}\ m}$

2) $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q \cdot q}{d^2} \qquad q = -1{,}6 \cdot 10^{-14}\ C \quad \wedge \quad d = 0{,}01\ m$

$\boxed{F = 2{,}3 \cdot 10^{-24}\ N}$

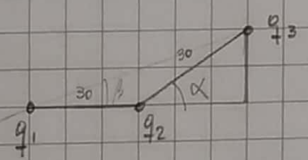
3) 

$\sin\alpha = 5/15 \qquad m = 0{,}5\ g = 0{,}0005\ Kg$

$\alpha = 19{,}5^\circ \qquad q_1 = q_2 = q$

$\begin{cases} F = T \sin\alpha \\ P = T \cos\alpha \end{cases} \qquad F = P\, tg\,\alpha = 1{,}73 \cdot 10^{-3}\ N$

$F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q\, q}{d^2} \rightsquigarrow \boxed{q = 4{,}39 \cdot 10^{-8}\ C}$

4) 

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = \dfrac{q_1}{4\pi\varepsilon_0} \left(\dfrac{q_2}{0{,}3^2} + \dfrac{q_3}{0{,}3^2 (1+\cos(\alpha))^2} ; \dfrac{q_3}{(0{,}3 \sin\alpha)^2} \right)$

$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} = \dfrac{q_2}{4\pi\varepsilon_0} \left(\dfrac{q_1}{0{,}3^2} + \dfrac{q_3}{(0{,}3 \cos\alpha)^2} ; \dfrac{q_3}{(0{,}3 \sin\alpha)^2} \right)$

$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} = \dfrac{q_3}{4\pi\varepsilon_0} \left(\dfrac{q_1}{(0{,}3(1+\cos\alpha))^2} + \dfrac{q_2}{(0{,}3\cos\alpha)^2} ; \dfrac{q_1}{(0{,}3 \sin\alpha)^2} + \dfrac{q_2}{(0{,}3 \sin\alpha)^2} \right)$

$tg\,\beta = \dfrac{\sin\alpha}{1+\cos\alpha} \qquad q_1 = 1\ \mu C \qquad q_2 = -2\ \mu C \qquad q_3 = 0{,}5\ \mu C$

Si $\alpha = 30^\circ$ $\quad \vec{F}_1 = (-0{,}185 ; 0{,}2)\ N$

$\vec{F}_2 = (-0{,}33 ; -0{,}4)\ N$

$\vec{F}_3 = (-0{,}119 ; -0{,}2)\ N$

Si $\alpha = 0^\circ$ $\quad \vec{F}_1 = (-0{,}187 ; 0)\ N$

$\vec{F}_2 = (-0{,}3 ; 0)\ N$

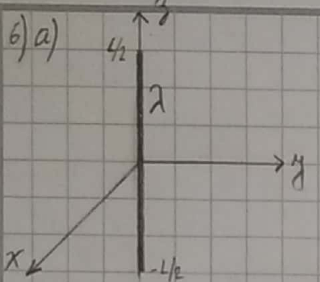
$\vec{F}_3 = (-0{,}0874 ; 0)\ N$

Si $\alpha = 90^\circ$ $\quad \vec{F}_1 = (\quad)$

$\vec{F}_2 = (\quad)$

$\vec{F}_3 = (\quad)$

6) a)



$d\ell' = \lambda' dz'$

$r = (x, y, z)$

$r' = (0, a, z')$

$r - r' = (x, y, z - z')$

$|r - r'|^3 = \left(x^2 + y^2 + (z - z')^2\right)^{3/2} = \left(a^2 + (z - z')^2\right)^{3/2}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{L'} \lambda' dz' \frac{(x, y, z - z')}{(a^2 + (z - z')^2)^{3/2}}$$

• $E_x = \frac{x\lambda'}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{dz'}{(a^2 + (z - z')^2)^{3/2}}$, $\begin{cases} u = z - z' \\ du = -dz' \end{cases}$, $E_x = \frac{-x\lambda'}{4\pi\varepsilon_0 a^2} \cdot \frac{z - z'}{(a^2 + (z - z')^2)^{1/2}}$

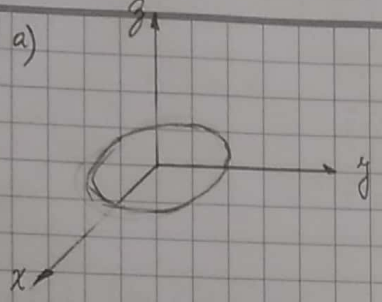
$$E_x = \frac{-x\lambda'}{4\pi\varepsilon_0 a^2} \left(\frac{z - L/2}{(a^2 + (z - L/2)^2)^{1/2}} - \frac{z + L/2}{(a^2 + (z + L/2)^2)^{1/2}} \right)$$

X	Y	Z
0,05	0,1	0,02

• $E_y = \frac{-y\lambda'}{4\pi\varepsilon_0 a^2} \left(\frac{z - L/2}{(a^2 + (z - L/2)^2)^{1/2}} - \frac{z + L/2}{(a^2 + (z + L/2)^2)^{1/2}} \right)$

• $E_z = \frac{\lambda'}{4\pi\varepsilon_0} \left(\frac{1}{(a^2 + (z - L/2)^2)^{1/2}} - \frac{1}{(a^2 + (z + L/2)^2)^{1/2}} \right)$

7) a)



$$d\ell' = R\,d\varphi'$$

$$r' = (R\cos\varphi; R\sin\varphi; 0)$$

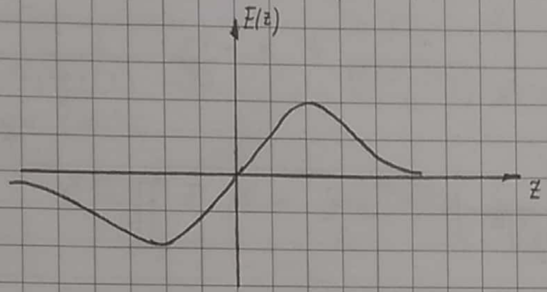
$$r = (0, 0, z)$$

$$(r - r') = (R\cos\varphi; R\sin\varphi; -z)$$

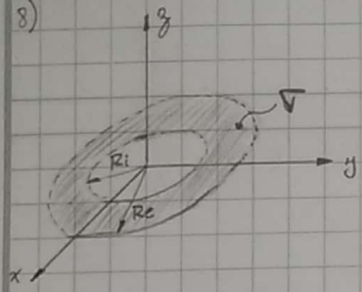
$$|r - r'|^3 = (R^2 + z^2)^{3/2}$$

$$\vec{E} = \frac{\lambda'}{4\pi\varepsilon_0}\int_0^{2\pi} R\,d\varphi' \frac{(-R\cos\varphi; -R\sin\varphi; +z)}{(R^2+z^2)^{3/2}} = \frac{\lambda' R}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} \left(-R\sin\varphi;\ R\cos\varphi;\ +z\varphi\right)\Big|_0^{2\pi}$$

$$\vec{E} = \frac{\lambda' R}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}} (0; 0; z.2\pi) = \frac{\lambda' R z}{2\varepsilon_0 (R^2+z^2)^{3/2}}\,\check{k}$$



a)



$$dq' = \sigma\, ds' \;,\quad ds' = r\, d\varphi\, dr \qquad |\vec{r} - \vec{r}'|^3 = (r^2 + z^2)^{3/2}$$

$$\vec{r} = (0, 0, z)$$

$$\vec{r}' = (r\cos\varphi ; r\sin\varphi ; 0)$$

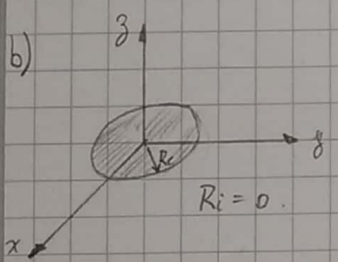
$$(\vec{r} - \vec{r}') = (-r\cos\varphi ; -r\sin\varphi ; z)$$

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_{R_i}^{R_e} r\, d\varphi\, dr \frac{(-r\cos\varphi ; -r\sin\varphi ; z)}{(r^2 + z^2)^{3/2}}$$

- $E_x = -\dfrac{\sigma}{4\pi\varepsilon_0}\displaystyle\int_0^{2\pi}\int_{R_i}^{R_e} \frac{r^2\cos\varphi}{(r^2 + z^2)^{3/2}}\, dr\, d\varphi = 0$
- $E_y = 0$
- $E_z = \dfrac{\sigma z}{4\pi\varepsilon_0}\displaystyle\int_0^{2\pi}\int_{R_i}^{R_e} \frac{r\, d\varphi\, dr}{(r^2 + z^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\int_{R_i}^{R_e} \frac{r}{(r^2 + z^2)^{3/2}}\, dr = -\frac{\sigma z}{2\varepsilon_0}\cdot \left.\frac{1}{(r^2 + z^2)^{1/2}}\right|_{R_i}^{R_e}$

$$E_z = \frac{\sigma \cdot z}{2\varepsilon_0}\left(\frac{1}{(R_i^2 + z^2)^{1/2}} - \frac{1}{(R_e^2 + z^2)^{1/2}}\right)$$

b)

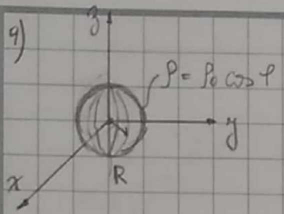


$R_i = 0$.

$$\vec{E} = \left(0, 0, \frac{-\sigma z}{2\varepsilon_0}\left(\frac{1}{(R_e^2 + z^2)^{1/2}} - \frac{1}{|z|}\right)\right)$$

d) $R_e \to \infty \leadsto \vec{E} = \left(0, 0, \dfrac{\sigma}{2\varepsilon_0}\dfrac{z}{|z|}\right)$

Signo de z

9) 

$dq' = \rho_0 \cos\varphi' \, dV' = \rho_0 \cos\varphi' \, dr_0 \, d\varphi \, d\theta \, (r^2 \sin\varphi)$

$\vec{r} = (x, y, z)$

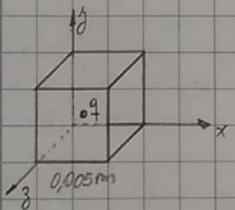
$\vec{r}\,' = (r \sin\theta \cos\varphi \,;\, r \sin\theta \sin\varphi \,;\, r\cos\theta)$

$(\vec{r} - \vec{r}\,') = (x - r\sin\theta\cos\varphi \,;\, y - r\sin\theta\sin\varphi \,;\, z - r\cos\theta)$

$|\vec{r} - \vec{r}\,'|^3 = \left[(x - r\sin\theta\cos\varphi)^2 + (y - r\sin\theta\sin\varphi)^2 + (z - r\cos\theta)^2\right]^{3/2}$

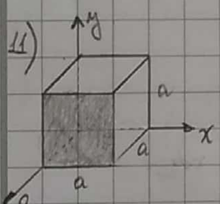
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \cdot \rho_0 \cos\varphi' \, r^2 \sin\varphi \, dr \, d\theta \, d\varphi}$$

10) Carga puntual: $q = 1 \cdot 10^{-6}$ C



$\Phi_E = \iint_S \vec{E} \cdot d\vec{s} = \dfrac{Q_{enc}}{\varepsilon_0} \rightsquigarrow Q_{enc} = q \rightsquigarrow \Phi_E = \dfrac{q}{\varepsilon_0} = 112.940,9 \, \dfrac{Nm^2}{C}$

Para cualquier superficie, el flujo será el mismo

11) 

- $\Phi_E = \iint_S \vec{E} \cdot d\vec{s} \overset{①}{=} \iiint_V \nabla \vec{E} \, dV \overset{②}{=} \dfrac{Q_{enc}}{\varepsilon_0}$
- $\nabla \vec{E} = \dfrac{\rho}{\varepsilon_0}$ (3) Siendo ρ: densidad de carga volumétrica

a) $\vec{E} = E_0 \, \check{i}$, Cálculo del gradiente: $\nabla \vec{E} = 0$, de (3): $\boxed{\rho = 0}$, de (2): $\boxed{Q_{enc} = 0}$, de (1) $\boxed{\Phi_E = 0}$

b) $\vec{E} = E_0 \, x \, \check{i}$, Cálculo del gradiente $\nabla \vec{E} = E_0$, de (3): $\boxed{\rho = E_0 \varepsilon_0}$ de (2) $\boxed{Q_{enc} = a^3 E_0 \varepsilon_0}$

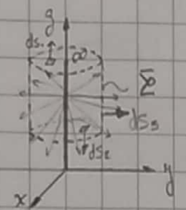
de (1) $\boxed{\Phi_E = a^3 \cdot E_0}$

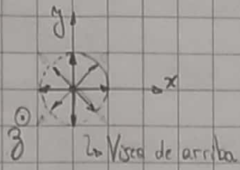
c) $\vec{E} = E_0 \, x^2 \, \check{i}$, Cálculo del gradiente: $\nabla \vec{E} = 2 E_0 x$, de (3) $\boxed{\rho = 2 E_0 x \varepsilon_0}$

de (2) $\iiint_V 2E_0 x = 2E_0 a^2 \cdot \dfrac{a^2}{2} = E_0 a^4 \rightsquigarrow \begin{cases} Q_{enc} = E_0 \varepsilon_0 a^4 \\ \Phi_E = E_0 a^4 \end{cases}$

d) $\vec{E} = E_0 \, y \, \check{i} + E_0 \, x \, \check{j}$, Gradiente: $\nabla \vec{E} = 0 \rightsquigarrow \boxed{\rho = 0}$ $\boxed{Q_{enc} = 0}$ $\boxed{\Phi_E = 0}$

13) a) Distribución lineal con densidad uniforme λ (infinita)

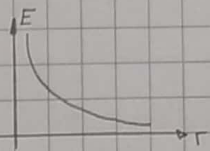




$$\frac{Q_{enc}}{\varepsilon_0} = \iint_\Sigma \vec{E}\, d\vec{s} = \iint_{S_1} E\check{r}\, ds_1\check{k} + \iint_{S_2} E\check{r}\, ds_2\check{k} + \iint_{S_3} E\check{r}\, ds_3\check{r}$$

$$\frac{\lambda z}{\varepsilon_0} = \int_0^{2\pi}\int_0^{z} E\, r\, dz\, d\varphi = E\, 2\pi r z$$

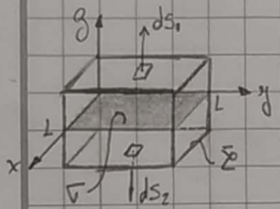
$$\boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\check{r}}$$



$E = f(r, \varphi, z)$

Previamente se supuso que para cualquier punto P del espacio, el campo es radial, anulándose las componentes z del mismo.

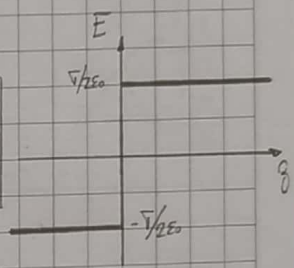
b) Distribución plana con densidad uniforme σ (infinita):



$$\iint_\Sigma E\, dz\, \check{k}\, d\vec{s}_i = \frac{Q_{enc}}{\varepsilon_0}, \quad (ds_3;\ ds_4;\ ds_5;\ ds_6): \iint \vec{E}\, d\vec{s} = 0$$

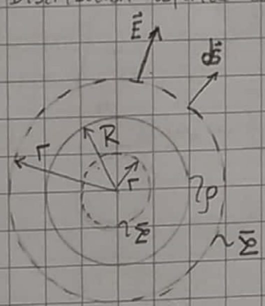
$$2E\int_0^L\int_0^L dx\, dy = \frac{\sigma L^2}{\varepsilon_0}$$

$$2E\cancel{L^2} = \frac{\sigma \cancel{L^2}}{\varepsilon_0} \rightsquigarrow \boxed{\vec{E} = \begin{cases} \sigma/2\varepsilon_0\, \check{k} & \text{si } z>0 \\ -\sigma/2\varepsilon_0\, \check{k} & \text{si } z<0 \end{cases}}$$



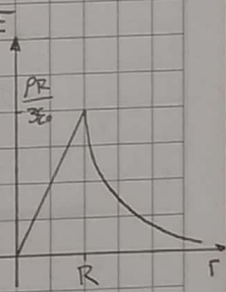
$E = f(x, y, z)$

c) Distribución esférica con densidad volumétrica de carga uniforme: (uniforme)



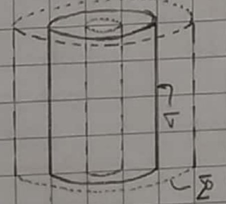
$$\iint_\Sigma E\check{r}\, ds\, \check{r} = \frac{Q_{enc}}{\varepsilon_0}$$

- $E\, 4\pi r^2 = \rho\, \frac{4\pi r^3}{3} \cdot \frac{1}{\varepsilon_0} \rightsquigarrow \vec{E} = \frac{\rho r}{3\varepsilon_0}\check{r} \quad r<R$
- $E\, \cancel{4\pi} r^2 = \rho\, \frac{\cancel{4}}{3}\cancel{\pi} R^3 \cdot \frac{1}{\varepsilon_0} \rightarrow \vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\check{r} \quad r>R$



$E = f(r, \varphi, \theta)$

d) Distribución cilíndrica con densidad superficial de carga uniforme. (infinita)

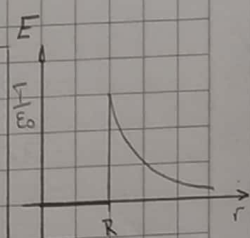


$$\iint_\Sigma E\check{r}\, ds\, \check{r} = \frac{Q_{enc}}{\varepsilon_0}, \quad \text{flujo nulo por las tapas dado que } E\check{r}\cdot ds\, \check{k} = 0$$

- $E\, 2\pi r z = 0$
- $E\, \cancel{2\pi} r z = \frac{\sigma\, \cancel{2\pi} R z}{\varepsilon_0}$

$\rightsquigarrow$ $\vec{E} = 0\, \check{r} \quad r<R$

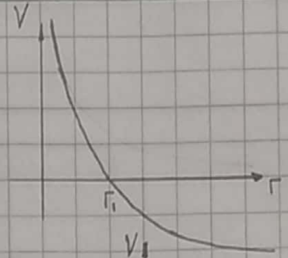
$\vec{E} = \frac{\sigma R}{r\varepsilon_0}\check{r} \quad r>R$



$E = f(r, \varphi, z)$

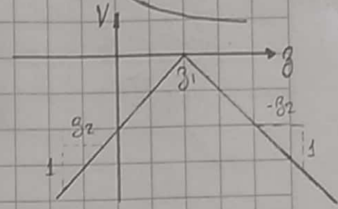
22) a) Sea $V(r_1) = 0$

$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi r \varepsilon_0}\, dr \rightsquigarrow V(r_2) = \frac{-\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{r}{r_1}\right)\right]$$



b) Sea $V(z_1)=0$

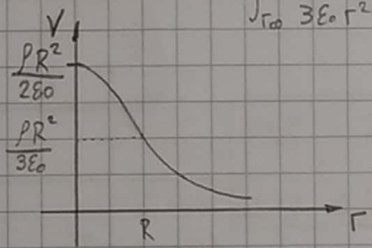
$$V(z_2) - V(z_1) = -\int_{z_1}^{z_2} E\, dz \rightsquigarrow V(z_2) = \begin{cases} \sigma/2\varepsilon_0 \cdot (z_2 - z_1), & z>0 \\ -\sigma/2\varepsilon_0 (z_2 - z_1), & z<0 \end{cases}$$



c) Distribución acotada de cargas $\rightsquigarrow V(\infty) = 0$

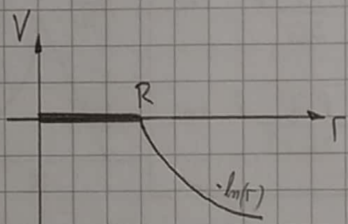
$$V(r) - V(\infty) = -\int_{r_\infty}^{r} E\, dr$$

$$V(r) = \begin{cases} -\int_R^r \frac{\rho r}{3\varepsilon_0} dr - \int_{r_\infty}^{R} \frac{\rho R^3}{3\varepsilon_0 r^2} dr = -\frac{\rho(r^2 - R^2)}{6\varepsilon_0} + \frac{\rho R^2}{3\varepsilon_0} - \cancelto{0}{\frac{\rho R^3}{3\varepsilon_0 r_\infty}} = -\frac{\rho r^2}{6\varepsilon_0} + \frac{\rho R^2}{2\varepsilon_0}, & r<R \\ -\int_{r_\infty}^{r} \frac{\rho R^3}{3\varepsilon_0 r^2} dr = \frac{\rho R^3}{3\varepsilon_0 r} - \cancelto{0}{\frac{\rho R^3}{3\varepsilon_0 r_\infty}} = \frac{\rho R^3}{3\varepsilon_0 r}, & r>R \end{cases}$$

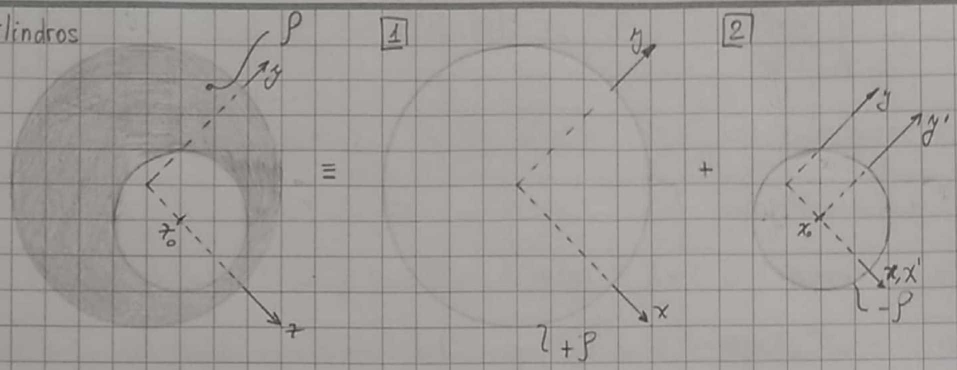


d) Distribución infinita de cargas $\rightsquigarrow V(R) = 0$

$$V(r) - V(R) = -\int_R^r E\, dr \rightsquigarrow V(r) = \begin{cases} -\int_R^r 0\, dr = 0 & r<R \\ -\int_R^r \frac{\sigma R}{r\varepsilon_0} dr = \frac{-\sigma R}{\varepsilon_0}\left[\ln(r) - \ln(R)\right] & r>R \end{cases}$$



15) Cilindros



Campo eléctrico generado por $\boxed{1}$

$$\iint_{\Sigma} \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0} \rightsquigarrow \begin{cases} E \cdot 2\pi \cancel{r}\, \cancel{g} = \dfrac{\rho\, \cancel{\pi} r^{\cancel{2}} \cancel{g}}{\varepsilon_0} \\ E\ 2\pi r\, g = \dfrac{\rho\, \cancel{\pi} R^2 \cancel{g}}{\varepsilon_0} \end{cases} \qquad \vec{E}_1 = \begin{cases} \dfrac{\rho r}{2\varepsilon_0}\, \check{r}, & r < R \\ \dfrac{\rho R_1^2}{2 r \varepsilon_0}\, \check{r}, & r > R \end{cases}$$

Campo eléctrico generado por $\boxed{2}$

$$\vec{E}_2 = \begin{cases} \dfrac{-\rho r}{2\varepsilon_0}\, \check{r} & r < R_2 \\ \dfrac{-\rho R_2^2}{2 r \varepsilon_0}\, \check{r} & r > R_2 \end{cases}$$

En coordenadas cartesianas

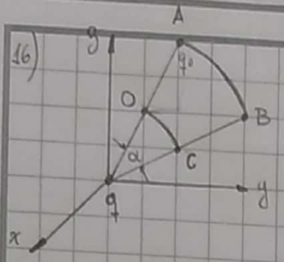
$$\vec{E}_1 = \begin{cases} \dfrac{\rho \overbrace{r \cos\theta}^{x}}{2\varepsilon_0}\, \check{i} + \dfrac{\rho \overbrace{r \sin\theta}^{y}}{2\varepsilon_0}\, \check{j} = \dfrac{\rho x}{2\varepsilon_0}\, \check{i} + \dfrac{\rho y}{2\varepsilon_0}\, \check{j} \\ \dots \end{cases}$$

$$\vec{E}_2 = \begin{cases} \dfrac{-\rho}{2\varepsilon_0} r' \cos\theta'\, \check{i} - \dfrac{\rho r'}{2\varepsilon_0} \sin\theta'\, \check{j} = \dfrac{-\rho}{2\varepsilon_0}(x - x_0)\, \check{i} - \dfrac{\rho}{2\varepsilon_0} y\, \check{j} \\ \dots \end{cases}$$

Superposición de efectos:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} \dfrac{\rho x_0}{2\varepsilon_0}\, \check{i} \\ \dots \end{cases}$$

16)



$$\left[W_{F_{ext}}^{A\to O} = q(V_O - V_A) = -q\int_A^O \vec{E}\, d\vec{r} = \frac{-q_0 q_1}{4\pi\varepsilon_0}\int_A^O \frac{1}{r^2}\, dr = \frac{q_0 q_1}{4\pi\varepsilon_0}\frac{1}{(r_O - r_A)} \right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} \quad ; \quad d\vec{r} = dr$$

Suponiendo que la carga se mueve cuasiestáticamente y de forma isoterma

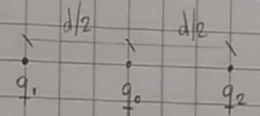
Luego, el trabajo que realiza la fuerza para llevar la carga de $A \to O$ es el mismo que si el camino fuera $A \to B \to C \to O$, ya que la carga se mueve por líneas equipotenciales al campo electrostático en los casos $A-B$ y $C-O$.

Se confirma que el campo electrostático es conservativo.

17) $W_{F_{ext}}^{\infty\to B} = q_0(V_B - V_A) = q_0\int_A^B -\vec{E}\cdot d\vec{r} = -\frac{q_0 q}{4\pi\varepsilon_0}\int_A^B \frac{1}{r^2}\, dr = \frac{q_0 q}{4\pi\varepsilon_0(r_B - r_A)} = \frac{q_0 q}{4\pi\varepsilon_0 r_B} - \cancel{\frac{q_0 q}{4\pi\varepsilon_0 r_A}}^{\,0}$ ($r_A \to \infty$)

$$\boxed{W_{F_{ext}}^{\infty\to B} = \frac{q_0 q}{4\pi\varepsilon_0 r_B}}$$

18) a)



$$W_{F_{ext}}^{\infty\to 0} = q_0\int_\infty^0 \vec{E}\cdot d\vec{r} = q_0(V(0) - \cancel{V(\infty)}^{\,0}) = q_0 V(0)$$

$$V(0) = \sum \frac{q_i}{4\pi\varepsilon_0}\cdot\frac{1}{d/2} = \frac{2}{4\pi\varepsilon_0 d}(q_1 + q_2) = \frac{(q_1+q_2)}{2\pi\varepsilon_0 d}$$

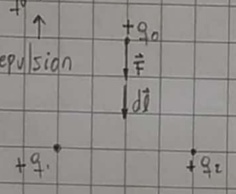
$$\boxed{W_{F_{ext}}^{\infty\to 0} = \frac{q_0(q_1+q_2)}{2\pi\varepsilon_0 d}}$$

b) $q_1 = q_2 > 0$ $\rightsquigarrow$ $\underbrace{W_{F_{ext}}^{\infty\to 0} = \frac{q_0 q_2}{\pi\varepsilon_0 d} > 0}$

$q_0 > 0$

↑

Repulsión



El trabajo es positivo ya que la fuerza se ejecuta en el sentido del movimiento

$q_1 = q_2 < 0$ $\rightsquigarrow$ $\underbrace{W_{F_{ext}}^{\infty\to 0} = \frac{q_0 q_2}{\pi\varepsilon_0 d} < 0}$

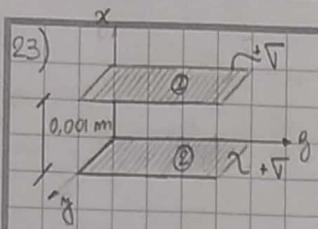
$q_0 > 0$

↑

Atracción

El trabajo es negativo ya que la fuerza se ejecuta en contra la atracción producida por las cargas para lograr un movimiento cuasiestático

23)



$\sigma = 50 \cdot 10^{-9}\ C/m^2$

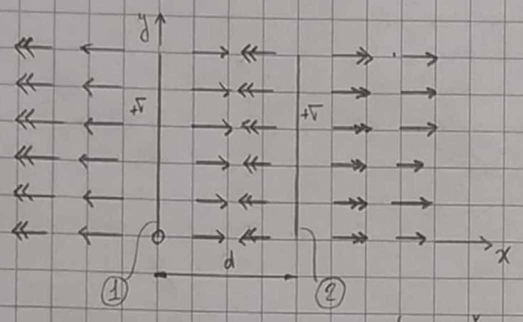
$\cancel{(\vec{r} - \vec{r}_1') = (-x'; -y'; z)}$ $\cancel{|\vec{r} - \vec{r}_1'|^3 = (x'^2 + y'^2 + z^2)^{3/2}}$

$\cancel{\vec{r} = (0, 0, z)}$ $\cancel{(\vec{r} - \vec{r}_2') = (-x'; -y'; z - 0{,}001) \quad |\vec{r} - \vec{r}_2'|^3 = (x'^2 + y'^2 + (z - 0{,}001)^2)^{3/2}}$

$\cancel{\vec{r}_1' = (x', y', 0)}$ $\cancel{\vec{r}_2' = (x'; y'; 0{,}001)}$

$\cancel{\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{S'} \sigma' ds' \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}}$ $\cancel{ds' = dx\, dy}$

$\cancel{\vec{E}_1 = \frac{\sigma}{4\pi\varepsilon_0} \int_0^x \int_0^y \frac{(-x, -y, z)}{(x^2 + y^2 + z^2)^{3/2}} dx\, dy} = \frac{\sigma}{4\pi\varepsilon_0} \Bigg(\frac{1}{(x^2 + y^2 + z^2)^{1/2}}$

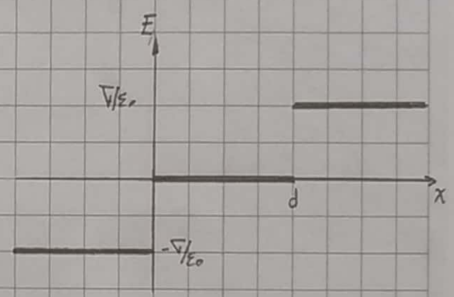


$E_1 = \frac{\sigma}{2\varepsilon_0}$ Suponiendo que los planos son infinitos

$E_2 = \frac{\sigma}{2\varepsilon_0}$

Por superposición de efectos:

$$\vec{E} = \begin{cases} -\frac{\sigma}{\varepsilon_0}\,\hat{i} & \text{si } x < 0 \\ 0\,\hat{i} & \text{si } x \in (0, d) \\ \frac{\sigma}{\varepsilon_0}\,\hat{i} & \text{si } x > d \end{cases}$$



Diferencia de potencial:

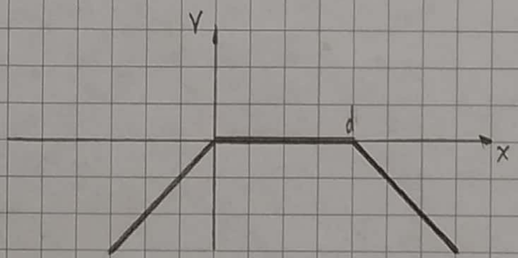
Referencia en $x = d \rightsquigarrow V(d) = 0$

$$V(0) - V(d) = V(0) = -\int_d^0 E\, dx = 0$$

$$V(x) - V(d) = V(x) = -\int_d^0 0\, dx - \int_0^x -\frac{\sigma}{\varepsilon_0}\, dx = \frac{\sigma}{\varepsilon_0} x$$

$$V(x) - V(d) = V(x) = -\int_d^x \frac{\sigma}{\varepsilon_0}\, dx = -\frac{\sigma}{\varepsilon_0}(x - d)$$

$$V(x) = \begin{cases} \sigma/\varepsilon_0 . x & \text{si } x < 0 \\ 0 & \text{si } x \in (0, d) \\ -\sigma/\varepsilon_0 . (x - d) & \text{si } x > d \end{cases}$$




Rivadavia

(b) $r_{ref}=\infty$

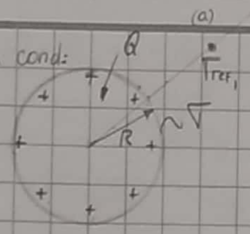
$dV = -\vec{E}\cdot d\vec{l}$

$\int_r^{r_{ef}} dV = -\int_r^{r_{ef}} \vec{E}\cdot d\vec{l}$

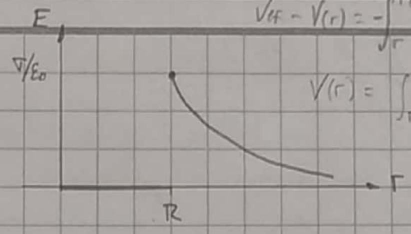
$V_{ef} - V(r) = -\int_r^{r_{ef}} \vec{E}\cdot d\vec{l}$

$V(r) = \int_r^{r_{ef}} \vec{E}\cdot d\vec{l}$

1) Esfera cond:



$$\vec{E} = \begin{cases} 0 & r<R \\ \dfrac{\sigma R^2}{\varepsilon_0 r^2}\,\check{r} & r \geq R \end{cases}$$



a)

Sea $V(2R) = 0$, $2R = r_{ef}$

$dV = -\vec{E}\cdot d\vec{l}$

$V(r_{ef}) - V(r_1) = -\int_{r_1}^{r_{ef}} E\, dl$

$V(r_1) - V_{ref} = \int_{r_1}^{r_{ef}} E\, dl$

$$V(r) - V_{ref} = \int_r^{r_{ef}} \vec{E}\cdot d\vec{l}$$

$\infty > r > 2R$

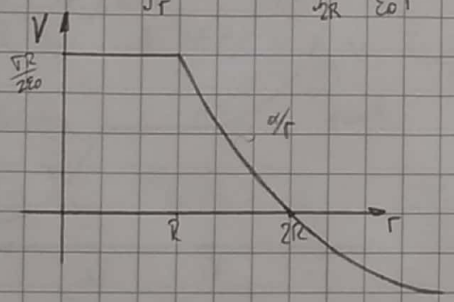
$$V(r) = -\int_{2R}^{r} \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = +\frac{\sigma R^2}{\varepsilon_0}\left(-\frac{1}{2R} + \frac{1}{r}\right) = -\frac{\sigma R}{\varepsilon_0 2} + \frac{\sigma R^2}{\varepsilon_0 r}$$

$R<r<2R$

$$V(r) = -\int_{2R}^{r} \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = +\frac{\sigma R^2}{\varepsilon_0}\left(\frac{1}{r} - \frac{1}{2R}\right) = \frac{\sigma R^2}{\varepsilon_0 r} - \frac{\sigma R}{\varepsilon_0 2}$$

$r<R$

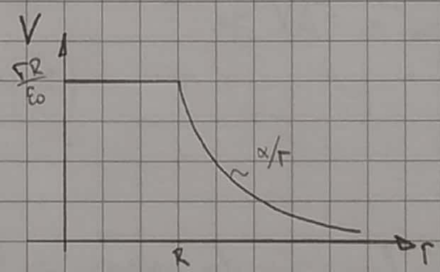
$$V(r) = +\int_r^R 0\, dr - \int_{2R}^{R} \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = -\frac{\sigma R^2}{\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right) = \frac{\sigma R}{\varepsilon_0 2}$$



b) Sea $r_{ef} = \infty \rightsquigarrow V_{ref} = V_\infty = 0$

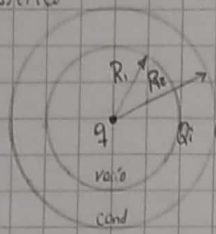
$$r<R:\ V(r) = \int_r^R 0\, dr + \int_R^\infty \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = +\frac{\sigma R^2}{\varepsilon_0 R} = \frac{\sigma R}{\varepsilon_0}$$

$$R<r<\infty:\ V(r) = \int_r^\infty \frac{\sigma R^2}{\varepsilon_0 r^2}\, dr = \frac{\sigma R^2}{\varepsilon_0 r}$$



$$\text{c) } W_{q_0}^{2R\to\infty} = q\left(V(\infty) - V(2R)\right) = -q\cdot\frac{\sigma R}{2\varepsilon_0}$$

2) Esférico



Si se plantea una superficie gaussiana entre R_1 y R_2 : $\Sigma \rightsquigarrow \oint_\Sigma \vec{E}\, d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$

↳ $E=0$ por ser conductor $\Rightarrow$ $Q_{enc} = 0 = q + Q_i \rightsquigarrow Q_i = -q$

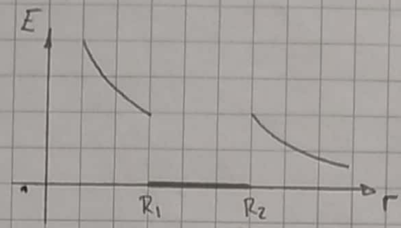
Por el principio de conservación de la carga, se tiene que $Q_2 = 0 = Q_e + Q_i \rightsquigarrow Q_e = q$

Para hallar el campo electrostático, se utilizará la ley de Gauss

Si $r < R_1$ $\quad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

Si $r \in (R_1;R_2)$ $\quad \vec{E} = 0\,\hat{r}$

Si $r > R_2$ $\quad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$



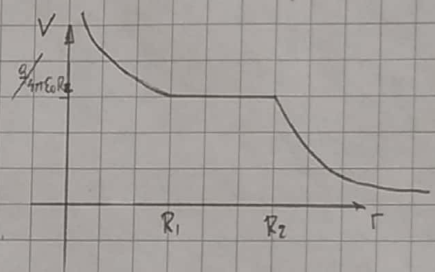
Cálculo del potencial: sea $V_{ref} = V_\infty = 0$ $\qquad dV = -\vec{E}\,d\vec{l} \rightsquigarrow V_{ref} - V(r) = -\int_r^{ref} \vec{E}\,d\vec{l} \rightsquigarrow V(r) = \int_r^{ref} \vec{E}\cdot d\vec{l}$

Si $r < R_1$ $\quad V(r) = \int_r^{R_1} \frac{q}{4\pi\varepsilon_0 r^2}\,dr + \int_{R_1}^{R_2} 0\,dr + \int_{R_2}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2}\,dr = -\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{r} + \frac{1}{\infty} - \frac{1}{R_2}\right)$

$$= \frac{q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

Si $r \in (R_1;R_2)$ $\quad V(r) = \int_r^{R_2} 0\,dr + \int_{R_2}^{\infty} \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R_2}$

Si $r > R_2$ $\quad V(r) = \int_r^{\infty} \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 r}$



$W_{Fex}^{\infty\to r} = \Delta V\, q_0$

b) Si el conductor está cargado con Q_2 $\Rightarrow$ $Q_i = -q$ y $Q_e = q + Q_2$

$$\vec{E} = \begin{cases} q/4\pi\varepsilon_0 r^2\,\hat{r} & r < R_1 \\ 0 & r \in (R_1;R_2) \\ (q+Q_2)/4\pi\varepsilon_0 r^2\,\hat{r} & r > R_2 \end{cases} \qquad V(r) = \begin{cases} q/4\pi\varepsilon_0 \cdot (1/r - 1/R_1) + (q+Q_2)/4\pi\varepsilon_0 R_2 & r < R_1 \\ (q+Q_2)/4\pi\varepsilon_0 R_2 & r \in (R_1;R_2) \\ (q+Q_2)/4\pi\varepsilon_0 r & r > R_2 \end{cases}$$

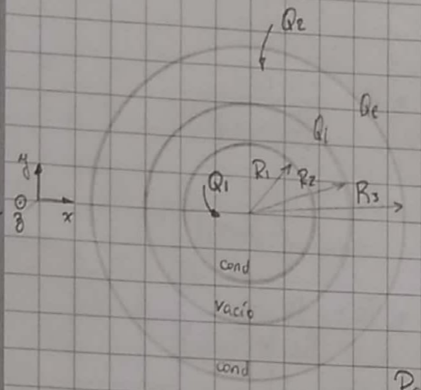
$W_{Fex}^{\infty\to r} = q_0 . \Delta V$

Plus: $Q_1 = \sigma_1 4\pi R_1^2$

$Q_i = \sigma_2 4\pi R_2^2$

$Q_e = \sigma_3 4\pi R_3^2$

3) Cilindros



Análisis de la distribución de carga:

. Como el cilindro 1 es conductor, Q_1 se distribuye uniformemente sobre toda la superficie de este. $\rightsquigarrow$ $\sigma_1 = Q_1/2\pi R_1 L$

. Tomando una sup gaussiana entre R_2 y R_3 resulta que, $Q_i = -Q_1$ dado que el campo electrostático en un conductor es nulo. $\rightsquigarrow$ $\sigma_2 = -Q_1/2\pi R_2 L$

. Por el principio de conservación de la carga, se tiene que $Q_2 = Q_i + Q_e$, por lo tanto resulta que $Q_e = Q_1 + Q_2$ $\rightsquigarrow$ $\sigma_3 = (Q_1+Q_2)/2\pi R_3 L$

Cálculo del campo electrostático:

Si $r < R_1$: $\quad \vec{E} = 0\,\hat{r}$

Si $r \in (R_1, R_2)$: $\quad E\,\cancel{2\pi} r \cancel{L} = \dfrac{Q_1}{\varepsilon_0} = \dfrac{\sigma_1\, \cancel{2\pi} R_1 \cancel{L}}{\varepsilon_0} \rightsquigarrow \vec{E} = \dfrac{\sigma_1 R_1}{\varepsilon_0 r}\,\hat{r}$

Si $r \in (R_2; R_3)$: $\quad E = 0\,\hat{r}$

Si $r > R_3$: $\quad E\,\cancel{2\pi} r \cancel{L} = \dfrac{Q_1 + Q_2}{\varepsilon_0} = \dfrac{\sigma_3\, \cancel{2\pi} R_3 \cancel{L}}{\varepsilon_0} \rightsquigarrow \vec{E} = \dfrac{\sigma_3 R_3}{\varepsilon_0 r}\,\hat{r}$

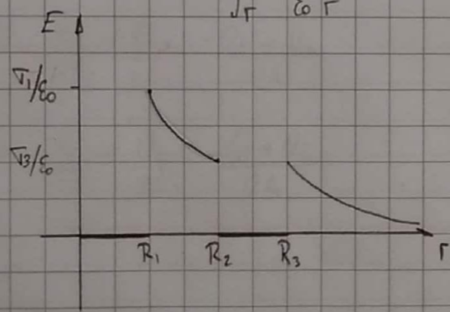
Cálculo del potencial eléctrico: sea $r_{ref} = R_3 \Rightarrow V_{R_3} = 0$ $\quad dV = -\vec{E}\cdot d\vec{l} \rightsquigarrow \int_r^{ref} dV = \int_r^{ref} -\vec{E}\cdot d\vec{l} \rightsquigarrow V(r) = \int_r^{ref} \vec{E}\cdot d\vec{l}$

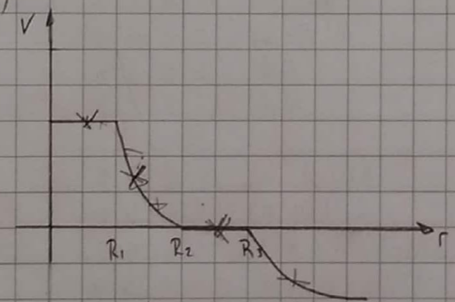
Si $r < R_1$: $\quad V(r) = \displaystyle\int_r^{R_1} 0\, dr + \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{\varepsilon_0 r}\, dr + \int_{R_2}^{R_3} 0\, dr = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)$

Si $r \in (R_1; R_2)$: $\quad V(r) = \displaystyle\int_r^{R_2} \frac{\sigma_1 R_1}{\varepsilon_0 r}\, dr + \int_{R_2}^{R_3} 0\, dr = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{r}\right)$

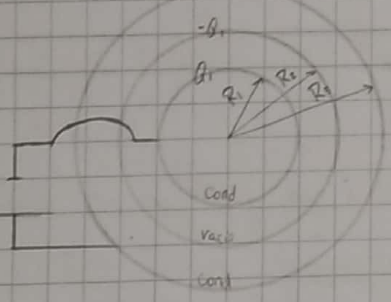
Si $r \in (R_2; R_3)$: $\quad V(r) = \displaystyle\int_r^{R_3} 0\, dr = 0$

Si $r > R_3$: $\quad V(r) = \displaystyle\int_r^{R_3} \frac{\sigma_3 R_3}{\varepsilon_0 r}\, dr = \frac{\sigma_3 R_3}{\varepsilon_0} \ln\left(\frac{R_3}{r}\right)$





4)



Carga $- \rightsquigarrow +$
Consist $+ \rightsquigarrow -$
E $+ \rightsquigarrow -$

$\frac{\sigma_1 R_1}{\varepsilon_0 r}$

$V(R_2) - V(R_1) = 10\,V$

$$-\int_{R_1}^{R_2} \frac{\sigma_1 R_1}{\varepsilon_0 r}\, dr = 10$$

$$\frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right) = -10 \rightsquigarrow * \; \sigma_1 = \frac{-10\,\varepsilon_0}{R_1 \ln(R_2/R_1)}$$

$$\frac{Q_1}{2\pi R_1 L}\, \frac{R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right) = -10 \rightsquigarrow \cdot \; Q_1 = \frac{-10\, 2\pi L \varepsilon_0}{\ln(R_2/R_1)}$$

$$* \; \sigma_2 = \frac{10\,\varepsilon_0}{R_2 \ln(R_2/R_1)}$$

$$\cdot \; Q_2 = \frac{10\, 2\pi L \varepsilon_0}{\ln(R_2/R_1)}$$

c)

$$\vec{E} = \begin{cases} 0 & r < R_1 \\ \dfrac{-10}{r \ln(R_2/R_1)}\, \hat{r} & r \in (R_1, R_2) \\ 0 & r > R_2 \end{cases}$$

d)

$$V(r) = \begin{cases} -\displaystyle\int_{R_1}^{r} \frac{-10}{r \ln(R_2/R_1)}\, dr = \frac{10}{\ln(R_2/R_1)} \cdot \ln\left(\frac{r}{R_1}\right) & , \; r \in (R_1, R_2) \\ -\displaystyle\int_{R_1}^{R_2} \frac{-10}{r \ln(R_2/R_1)}\, dr - \int_{R_2}^{r} 0\, dr = 10 & , \; r > R_2 \end{cases}$$

Siendo $r_{ef} = R_1$, $V(R_1) = 0$

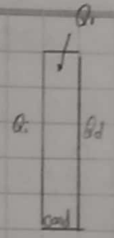
$-\int_{R_2}^{r} 0\, dr$

e) $V(R_3) - V(R_1) = -5 = -\displaystyle\int_{R_1}^{R_2} \frac{\sigma_1 R_1}{\varepsilon_0 r}\, dr = \frac{-\sigma_1 R_1}{\varepsilon_0} \ln(R_2/R_1) \rightsquigarrow \sigma_1 = \frac{5\,\varepsilon_0}{R_1 \ln(R_2/R_1)}$

$V(R_1) - V(R_3) = 5$

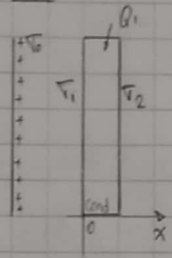
$$\vec{E} = \begin{cases} 0 & r < R_1 \\ \dfrac{5}{r \ln(R_2/R_1)}\, \hat{r} & r \in (R_1; R_2) \\ 0 & r > R_2 \end{cases}$$

$$V(r) = \begin{cases} 0 & \text{si } r < R_1 \\ -\displaystyle\int_{R_1}^{r} \frac{5}{r \ln(R_2/R_1)}\, dr = \frac{-5}{\ln(R_2/R_1)} \ln(r/R_1) & r \in (R_1, R_2) \\ -5 & r > R_2 \end{cases}$$

5) 

$$\vec{E} = \begin{cases} \sigma/2\varepsilon_0 \;\hat{i} & x>0 \\ -\sigma/2\varepsilon_0 \;\hat{i} & x<0 \end{cases}$$

$$\sigma = \frac{Q_1}{2L^2} \quad , \quad Q_i = Q_d = \frac{Q_1}{2} \rightsquigarrow Q_1 = \sigma_1 A + \sigma_2 A \qquad (1)$$

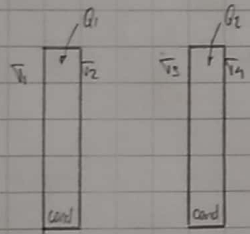
b) 

Al se suponen tres distribuciones planas infinitas, se tiene que, respecto de $\vec{P} = d/2\,\hat{i}$

$$\begin{cases} E_0 = \sigma_0/2\varepsilon_0 \\ E_1 = \sigma_1/2\varepsilon_0 \\ E_2 = -\sigma_2/2\varepsilon_0 \end{cases} \rightsquigarrow \text{Superposición de efecto } E(P) = \frac{1}{2\varepsilon_0}\left(\sigma_0 + \sigma_1 - \sigma_2\right) = 0 \qquad (\text{II})$$

$$\begin{cases} Q/A = \sigma_1 + \sigma_2 \\ 0 = \sigma_0 + \sigma_1 - \sigma_2 \end{cases} \qquad Q/A = \sigma_1 + \sigma_0 + \sigma_1 \rightsquigarrow \sigma_1 = \left(\frac{Q}{A} - \sigma_0\right)\cdot\frac{1}{2} = \frac{Q}{2A} - \frac{\sigma_0}{2}$$

$$\sigma_2 = \sigma_0 + \frac{(Q/A - \sigma_0)}{2} = \frac{Q}{2A} + \frac{\sigma_0}{2}$$

c) 

$$Q_1/A = \sigma_1 + \sigma_2 \qquad (\text{I}) \qquad\qquad E(P_1) = 0 = \frac{1}{2\varepsilon_0}\left(\sigma_1 - \sigma_2 - \sigma_3 - \sigma_4\right) \qquad (\text{III})$$

$$Q_2/A = \sigma_3 + \sigma_4 \qquad (\text{II}) \qquad\qquad E(P_2) = 0 = \frac{1}{2\varepsilon_0}\left(\sigma_1 + \sigma_2 + \sigma_3 - \sigma_4\right) \qquad (\text{IV})$$

(IV) en (III) : $\cancel{\sigma_1} - \sigma_2 - \sigma_3 - \cancel{\sigma_1} - \sigma_2 - \sigma_3 = 0 \qquad \sigma_2 = \sigma_3$

$Q_1/A = \sigma_1 + \sigma_2 \qquad Q_1/A - Q_2/A + \sigma_4 = \sigma_1 \qquad Q_1/A - Q_2/A + \cancel{\sigma_4} - \sigma_2 - \sigma_3 - \cancel{\sigma_4} = 0$

$Q_2/A = \sigma_2 + \sigma_4 \rightsquigarrow \sigma_2 = Q_2/A - \sigma_4 \qquad \sigma_2 = Q_1/A - Q_2/A - \sigma_3$

$$\sigma_1 = \frac{Q_2}{2A} + \frac{Q_1}{2A} = \frac{Q_1 + Q_2}{2A}$$

$$\sigma_2 = \frac{Q_1 - Q_2}{2A} = \frac{Q_1 - Q_2}{2A}$$

$$\sigma_3 = \frac{Q_2}{2A} - \frac{Q_1}{2A} = \frac{-Q_1 + Q_2}{2A}$$

$$\sigma_4 = \frac{Q_1}{2A} + \frac{Q_2}{2A} = \frac{Q_1 + Q_2}{2A}$$

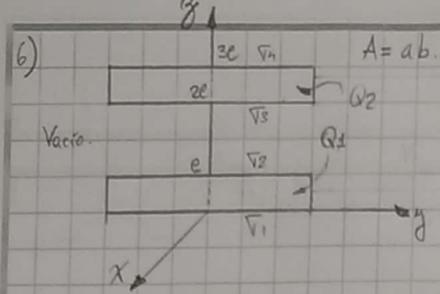
$$\left(\begin{array}{cccc|c} -1 & +1 & +1 & +1 & 0 \\ -1 & -1 & -1 & +1 & 0 \\ 1 & 1 & 0 & 0 & Q_1/A \\ 0 & 0 & 1 & 1 & Q_2/A \end{array}\right)$$

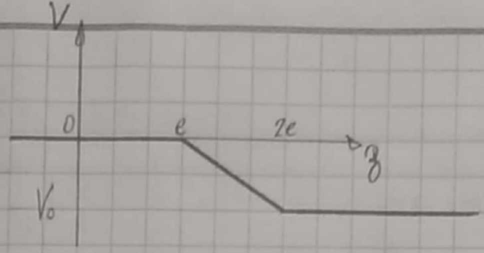
$$\begin{array}{cccc|c} 0 & 2 & 1 & 1 & Q_1/A \\ 0 & 0 & -1 & 1 & Q_1/A \\ 1 & 1 & 0 & 0 & Q_1/A \\ 0 & 0 & 1 & 1 & Q_2/A \end{array}$$

$$\begin{array}{cccc|c} 0 & 2 & 0 & 0 & (Q_1 - Q_2)/A \\ 0 & 0 & 0 & 2 & (Q_1 + Q_2)/A \\ 1 & 1 & 0 & 0 & Q_1/A \\ 0 & 0 & 1 & 1 & Q_2/A \end{array}$$

$$\begin{array}{cccc|c} 0 & 1 & 0 & 0 & (Q_1 - Q_2)/2A \\ 0 & 0 & 0 & 1 & (Q_1 + Q_2)/2A \\ 1 & 0 & 0 & 0 & Q_1/A - (Q_1 - Q_2)/2A \\ 0 & 0 & 1 & 0 & Q_2/A - (Q_1 + Q_2)/2A \end{array}$$

6)





$$V(z) = \begin{cases} 0 & z < e \\ -\alpha z & e < z < 2e \\ V_0 & z > 2e \end{cases}$$

a) $\int_0^{2e} \vec{E} \cdot d\vec{z} = \int_0^e 0 \, dz + \int_e^{2e} E \, dz = -\Delta V = -(-V_0) = V_0$

$-\alpha z \Big|^{2e} = V_0$

$\alpha \, 2e = V_0$

$$\vec{E} = \begin{cases} 0 & z < e \\ E_0 & e < z < 2e \\ 0 & z > 2e \end{cases} \iff$$

$Q_1 = Q$

$Q_2 = -Q$

$$\begin{cases} \sigma_1 = 0 \\ \sigma_2 = Q/A \\ \sigma_3 = -Q/A \\ \sigma_4 = 0 \end{cases}$$

$E = -\text{grad}(V) = -(V'_x ; V'_y ; V'_z) = (0, 0, -V'_z)$

$\vec{E} = -V'_z \, \check{z}$, $-V'_z = -\frac{\Delta V}{\Delta z} = \frac{(-V_0) - 0}{2e - e} = +\frac{V_0}{e}$

$\vec{E} = \frac{V_0}{e} \check{z}$

8) ε_{r_1} | ε_{r_2}

$(0,1;0)$

$(-0,1;0) = A$ $C = (0,2;0,05)$ $B = (0,2;0)$

$\varepsilon_{r_1} = 3,5$ $V_A - V_B = 200\,V$ Se consideran los

$\varepsilon_{r_2} = 6,25$ $V_B - V_C = 50\,V$ campos uniformes

Hallar E, D, P

• $$V_A - V_B = -\int_B^A \bar{E}\cdot d\bar{l} = \int_A^B E_x\,dx = \int_A^0 E_{x_1}\,dx + \int_0^B E_{x_2}\,dx = E_{x_1}\cdot 0,1 + E_{x_2}\,0,2 = 200 \quad (1)$$

De la cond de frontera, tenemos que $D_{N_1} = D_{N_2} \Rightarrow D_{x_1} = D_{x_2}$ dado que $\sigma_{interfase} = 0$

$$\vec{D} = \varepsilon\vec{E} \Rightarrow D_x = \varepsilon E_x \Rightarrow \varepsilon_1 E_{x_1} = \varepsilon_2 E_{x_2} \quad (2)$$

(1) y (2)

$$\frac{\varepsilon_2}{\varepsilon_1} E_{x_2}\, 0,1 + E_{x_2}\,0,2 = 200$$

$$E_{x_2}\left[\frac{\cancel{\varepsilon_0}\varepsilon_{r_2}}{\cancel{\varepsilon_0}\varepsilon_{r_1}}(0,1) + 0,2\right] = 200 \qquad \begin{cases} E_{x_2} = 528,3\ N/C \\ E_{x_1} = 943,4\ N/C \end{cases}$$

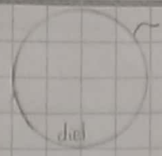
• $$V_B - V_C = -\int_C^B \vec{E}\cdot d\vec{l} = \int_B^C E_{y_2}\cdot dy = E_{y_2}\cdot 0,05 = 50 \rightsquigarrow E_{y_2} = 1000\ N/C$$

De la cond de frontera, tenemos que $E_{y_1} = E_{y_2} \Rightarrow E_{y_1} = 1000\ N/C$

$$\vec{E_1} = (943,4\,;\,1000)\ N/C \qquad \vec{D_1} = (2,92.10^{-8}\,;\,3,1\,10^{-8})\ C/m^2$$
$$\vec{E_2} = (528,3\,;\,1000)\ N/C \Longrightarrow \vec{D_2} = (2,92.10^{-8}\,;\,5,53.10^{-8})\ C/m^2$$

$$\Longrightarrow \vec{P_1} = (2,08.10^{-8}\,;\,2,21.10^{-8})\ C/m^2 \qquad \vec{D} = \varepsilon_0\varepsilon_r\vec{E}$$
$$\vec{P_2} = (2,45.10^{-8}\,;\,4,64.10^{-8})\ C/m^2 \qquad \vec{P} = \vec{D} - \varepsilon_0\vec{E}$$

9) Esfera dieléctrica: $R = 10^{-5}\ m$ $\quad Q = 2\ 10^{-12}\ C$ $\quad \varepsilon_r = 2{,}8$ $\quad \rho = 477{,}5\ C/m^3$



a) Ley de Gauss generalizada:

$$* \oiint_{\Sigma} \vec{D} \cdot d\vec{s} = Q_L \rightarrow \bullet\ \vec{D} = \frac{\rho r}{3}\ \check{r} \quad \text{si } r<R$$

$$\bullet\ \vec{E} = \frac{\rho r}{\varepsilon_0 \varepsilon_r 3}\ \check{r} \quad \text{si } r<R$$

$$\bullet\ \vec{P} = \frac{\rho r}{3}\left(1 - \frac{1}{\varepsilon_r}\right)\check{r} \quad r<R$$

Ley de Gauss:

$$* \oiint_{\Sigma} \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0} \rightarrow \bullet\ \vec{E} = \frac{\rho R^3}{r^2 3 \varepsilon_0}\ \check{r} \quad r>R$$

Susceptibilidad eléctrica del vacío = 0

$$\varepsilon_r = 1 + \chi = 1$$

$$\bullet\ \vec{D} = \frac{\varepsilon_r \rho R^3}{r^2 . 3}\ \check{r} \quad r>R$$

$$\vec{D} = \frac{\rho R^3}{r^2 3}\ \check{r} \quad r>R$$

$$\bullet\ \vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \vec{0} \quad r>R$$

Diferencia de potencial $\quad ref = \infty \ ,\ V_\infty = 0$

$$V(r) = + \int_r^R \frac{\rho r}{\varepsilon_0 \varepsilon_r 3}\, dr + \int_R^\infty \frac{\rho R^3}{r^2 3 \varepsilon_0} . dr = \frac{\rho}{6 \varepsilon_0 \varepsilon_r}(-r^2 + R^2) + \frac{\rho R^2}{3 \varepsilon_0} \qquad r<R$$

$$V(r) = + \int_r^\infty \frac{\rho R^3}{r^2 3 \varepsilon_0}\, dr = \frac{\rho R^3}{r 3 \varepsilon_0} \qquad r>R$$

b) $\rho_L = ?$ $\quad \rho_P = ?$ $\quad \sigma_P = ?$

$$\bullet\ \rho_L = \frac{Q}{\frac{4}{3}\pi R^3} = 477{,}5\ C/m^3 \quad , \quad \bullet\ \sigma_P = \vec{P} . \check{\eta} = \frac{\rho r}{3}(1 - 1/\varepsilon_r)$$

$$\bullet\ \rho_P = -\bar{\nabla} . \bar{P} = -\frac{1}{r^2}\frac{\partial (r^2 . P_r)}{\partial r} = \frac{-1}{\cancel{r^2}} \rho \cancel{r^2}(1 - 1/\varepsilon_r) = -\rho(1 - 1/\varepsilon_r)$$

↑ solo radial

Verifico $\quad Q_P = 0 = \oiint_S \sigma_P\, ds + \iiint_{Vol} \rho_P\, dVol = \frac{\rho}{3}(1 - 1/\varepsilon_r)\, 4\pi R^2 R - \rho \frac{4}{3}\pi R^3 (1 - 1/\varepsilon_r) = 0$ ✓

10) Esfera dieléctrica cargada superficialmente con $\sigma_L = \frac{Q}{4\pi R^2}$

Analisis dentro de la esfera: $r < R$

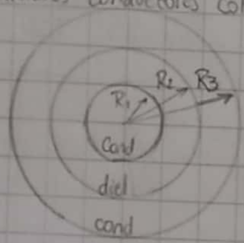
Ley generalizada de Gauss: $\iint_S \vec{D}\cdot d\vec{s} = Q_L = 0 \Rightarrow \vec{D} = \vec{0} \quad \vec{E} = 0 \quad \vec{P} = 0.$

Analisis fuera de la esfera $r > R$

Ley de Gauss: $\iint_S \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$, $\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\check{r}$, $\vec{D} = \frac{\sigma R^2 \varepsilon_r}{r^2}\check{r}$, $\vec{P} = \vec{0}.$

$\sigma_P = 0$, $\rho_P = 0$

12.3) Cilindros conductores colineales



$r < R_1 \quad \vec{E} = \vec{0} \quad \vec{D} = \vec{0} \quad \vec{P} = \vec{0}$

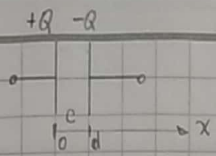
$R_1 < r < R_2 \quad D\,2\pi r L = \sigma_1 2\pi R_1 L \rightsquigarrow \vec{D} = \frac{\sigma_1 R_1}{r}\check{r}$, $\vec{E} = \frac{\sigma_1 R_1}{\varepsilon_0 \varepsilon_r r}\check{r}$

$\vec{P} = \frac{\sigma_1 R_1}{r}\left(1 - 1/\varepsilon_r\right)\check{r}$

$R_2 < r < R_3 \quad \vec{E} = 0 \quad \vec{D} = 0 \quad \vec{P} = 0$

$R_3 > r \quad \vec{E} = \frac{Q_1 + Q_2}{\varepsilon_0 2\pi r L}\check{r}$, $\vec{D} = \frac{\varepsilon_r}{2\pi r L}(Q_1 + Q_2)\check{r} = \frac{Q_1 + Q_2}{2\pi r L}\check{r}$, $\vec{P} = \vec{0}$

13) a) Capacitor de placas paralelas:



Sin dieléctrico:

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\check{i} \quad 0<x<d \quad , \quad V(x) = \int_x^d \frac{\sigma}{\varepsilon_0}\,dx = \frac{\sigma}{\varepsilon_0}(d-x) \Rightarrow \Delta V_P = \frac{\sigma}{\varepsilon_0} d \rightsquigarrow \text{Dif de pot entre placas}$$

$$C = \frac{Q}{\Delta V_P} = \frac{Q}{\frac{\sigma d}{\varepsilon_0}} = \frac{Q\varepsilon_0}{\sigma d} = \frac{Q\varepsilon_0}{\frac{Q}{S}d} = \frac{S\varepsilon_0}{d} \rightarrow C = \frac{S\varepsilon_0}{d}$$

$$\left[U = \frac{Q\,\Delta V_P}{2} = \frac{Q\,\sigma d}{2\varepsilon_0} = \frac{Q^2 d}{2S\varepsilon_0} \right]$$

Con dieléctrico:

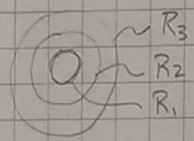
$$\iint_S \vec{D}\cdot d\vec{s} = Q_L = 2D\cancel{S} = \sigma \cancel{S} \rightsquigarrow \vec{D}_1 = \frac{\sigma}{2}\check{i} \qquad \vec{D}_2 = \frac{\sigma}{2}\check{i} \qquad x\in(0,d) \Rightarrow \vec{D} = \sigma\check{i} \quad x\in(0;d)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0\varepsilon_r}\check{i} \quad x\in(0;d) \qquad \vec{P} = \sigma\left(1 - 1/\varepsilon_r\right) \qquad C = \frac{Q}{\Delta V_P} = \frac{\cancel{\sigma}S.\varepsilon_0\varepsilon_r}{\cancel{\sigma}d} = \frac{\varepsilon_0\varepsilon_r S}{d}$$

$$V(x) = \int_x^d \frac{\sigma}{\varepsilon_0\varepsilon_r}\,dx = \frac{\sigma}{\varepsilon_0\varepsilon_r}(d-x) \qquad \Delta V_P = \frac{\sigma d}{\varepsilon_0\varepsilon_r}$$

$$\left[U = \frac{Q\,\Delta V_P}{2} = \sigma S\cdot\frac{\sigma d}{\varepsilon_0\varepsilon_r}\cdot\frac{1}{2} = \frac{\sigma^2 d S}{2\varepsilon_0\varepsilon_r} \right]$$

b) Capacitor cilíndrico:



Sin dieléctrico:

$$\vec{E} = \begin{cases} \dfrac{Q}{\varepsilon_0 2\pi r L}\check{r} & R_1<r<R_2 \\ 0 & \text{Otro caso} \end{cases}$$

$$\Delta V_P = \int_{R_1}^{R_2} \frac{Q}{\varepsilon_0 2\pi r L}\,dr = \frac{Q}{\varepsilon_0 2\pi L}\ln\left(\frac{R_2}{R_1}\right) \qquad U = \frac{Q\Delta V}{2} = \frac{Q^2 \ln(R_2/R_1)}{4\pi\varepsilon_0 L}$$

$$C = \frac{Q_{ext}}{\Delta V_P} = \frac{Q\,\varepsilon_0 2\pi L}{Q.\ln(R_2/R_1)} = \frac{\varepsilon_0 2\pi L}{\ln(R_2/R_1)}$$

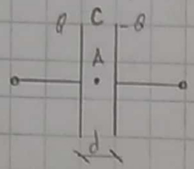
Con dieléctrico:

$$\vec{D} = \frac{Q}{2\pi r L}\check{r} \qquad \vec{E} = \frac{Q}{2\pi r L\varepsilon_0\varepsilon_r}\check{r} \qquad \vec{P} = \frac{Q}{2\pi r L}\left(1 - 1/\varepsilon_r\right) \qquad r\in\left(R_1;R_2\right)$$

$$\Delta V_P = \frac{Q}{2\pi L\varepsilon_0\varepsilon_r}\ln\left(R_2/R_1\right) \qquad C = \frac{Q}{\Delta V_P} = \frac{Q\,2\pi L\varepsilon_0\varepsilon_r}{Q\ln(R_2/R_1)} = \frac{2\pi L\varepsilon_0\varepsilon_r}{\ln(R_2/R_1)}$$

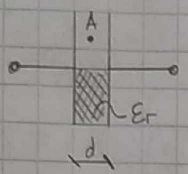
$$U = \frac{Q\,\Delta V}{2} = \frac{Q^2\ln(R_2/R_1)}{4\pi L\varepsilon_0\varepsilon_r}$$

14) Capacitor plano: se desprecian efectos de borde



$\vec{E}_A = \frac{Q}{S\varepsilon_0}\,\hat{i}$; $\Delta V = -\int_0^d \vec{E}\cdot d\vec{l} = -E\,d$

a) Con dielectrico dispuesto en la mitad inferior:



$\sigma_{interfase} = 0 \Rightarrow E_{t_1} = E_{t_2}$ (1)

$E_1 = \frac{\sigma_1}{\varepsilon_0} = \frac{Q_1}{S_1\varepsilon_0} = \frac{Q_1}{\frac{S}{2}\varepsilon_0}$ (2)

$D_2 = \frac{Q_2}{S_2} = \frac{Q_2}{\frac{S}{2}} \rightsquigarrow E_2 = \frac{2Q_2}{S\varepsilon_0\varepsilon_r}$ (3)

(2) y (3) a (1)

$\frac{2Q_1}{\cancel{S\varepsilon_0}} = \frac{2Q_2}{\cancel{S\varepsilon_0}\varepsilon_r} \rightsquigarrow Q_1 = \frac{Q_2}{\varepsilon_r}$

$Q = Q_1 + Q_2 \to Q_1 = \frac{Q - Q_1}{\varepsilon_r}$

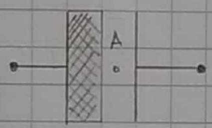
$Q = Q_1(1+\varepsilon_r)$

$Q_1 = Q/(1+\varepsilon_r)$

En el punto A: $E_A = \frac{Q_1}{S_1\varepsilon_0} = E_1$

$$\frac{E_{A\,desp}}{E_{A\,antes}} = \frac{Q_1}{S_1\varepsilon_0}\cdot\frac{S\varepsilon_0}{Q} = \frac{\cancel{Q_1}}{\frac{\cancel{S}}{2}\cancel{\varepsilon_0}}\cdot\frac{\cancel{S}\cancel{\varepsilon_0}}{\cancel{Q_1}(1+\varepsilon_r)} = \frac{2}{1+\varepsilon_r}$$

b)



Con dielectrico en la mitad izquierda.

$D_1 = \frac{Q}{S}$ $\quad E_1 = \frac{Q}{S\varepsilon_0\varepsilon_{r_1}}$ $\quad$ Cond de frontera $D_{1N} = D_{2N} \rightsquigarrow D_1 = D_2 \rightsquigarrow E_1 = E_2$

$E_2 = \frac{Q}{S\varepsilon_0\varepsilon_{r_2}} = \frac{Q}{S\varepsilon_0} = E_A$ $\qquad \frac{E_{A\,desp}}{E_{A\,antes}} = 1$

15) $\Delta V_p = V_0 = -\int_0^d \vec{E}_A\cdot d\vec{l} = -E_A d \rightsquigarrow E_A = -V_0/d$

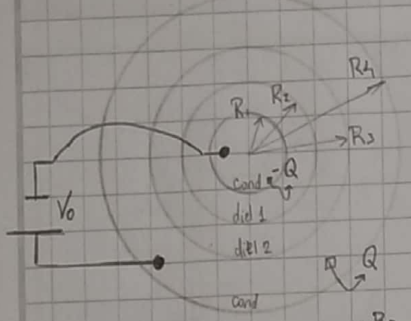
Caso a: $\Delta V_P = V_0 = -\int_0^d \bar{E}_A\cdot d\bar{l} = -E_A d$ $\qquad$ El campo no varía

Caso b: $\Delta V_P = V_0 = -\int_0^{d/2} \vec{E}_1\cdot d\vec{l} - \int_{d/2}^{d} \vec{E}_2\cdot d\vec{l} = -E_1\, d/2 - E_2\, d/2$

$E_2 = -E_1 - \frac{V_0 2}{d}$ $\qquad D_1 = \frac{Q}{S}$ $\qquad E_1 = \frac{Q}{S\varepsilon_0\varepsilon_r}$

$E_2 = \frac{-Q}{S\varepsilon_0\varepsilon_r} - \frac{V_0 2}{d} = \frac{-V_0}{d}$

16) Distribución cilíndrica.



1) $Q_P = 0 = \iint_{S_2} \sigma_{P_{12}}\, ds + \iint_{S_3} \sigma_{P_2}\, ds + \iint_{S_2} \sigma_{P_{21}}\, ds + \iiint_{Vol_1} \rho_{P_1}\, dVol + \iiint_{Vol_2} \rho_{P_2}\, dVol.$

2) Σ cil $\in (R_1;R_2) \rightsquigarrow D_1 2\pi r L = \sigma_{L_1} 2\pi R_1 L \rightsquigarrow D_1 = \dfrac{\sigma_{L_1} R_1}{r}$

$$E_1 = \frac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_1} r} \qquad R_1 < r < R_2$$

$$D_2 = \frac{\sigma_{L_1} R_1}{r} \wedge E_2 = \frac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_2} r} \qquad R_2 < r < R_3$$

4) $V(R_3) - V(R_1) = V_0 = -\int_{R_1}^{R_3} \vec{E}\cdot d\vec{l} = \int_{R_3}^{R_2} E_2\, dr + \int_{R_2}^{R_1} E_1\, dr \rightsquigarrow \int_{R_3}^{R_2} E_2\, dr = V_0 + \dfrac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_1}} \ln\left(\dfrac{R_2}{R_1}\right)$

$$\frac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_2}} \ln\left(\frac{R_2}{R_3}\right) = V_0 + \frac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_1}} \ln \frac{R_2}{R_1}$$

$$\frac{\sigma_{L_1} R_1}{\varepsilon_0}\left(\frac{1}{\varepsilon_{r_2}} \ln \frac{R_2}{R_3} - \frac{1}{\varepsilon_{r_1}} \ln\left(\frac{R_2}{R_1}\right)\right) = V_0$$

$$\sigma_{L_1} = \frac{V_0\, \varepsilon_0}{R_1\left(\frac{1}{\varepsilon_{r_2}} \ln \frac{R_2}{R_3} - \frac{1}{\varepsilon_{r_1}} \ln \frac{R_2}{R_1}\right)}$$

5) $\vec{P}_1 = \vec{D}_1 - \varepsilon_0 \vec{E}_1 = \dfrac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_1}\right) \check{r} \qquad r \in (R_1; R_2)$

$\vec{P}_2 = \vec{D}_2 - \varepsilon_0 \vec{E}_2 = \dfrac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_2}\right) \check{r} \qquad r \in (R_2; R_3)$

$$\sigma_{P_1} = \vec{P}_1(-\check{r}) = -\frac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_1}\right)\Big|_{R_1} = -\sigma_{L_1}\left(1 - 1/\varepsilon_{r_1}\right)$$

$$\sigma_{P_{12}} = \vec{P}_1(+\check{r}) = +\frac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_1}\right)\Big|_{R_2} = +\sigma_{L_1} \frac{R_1}{R_2}\left(1 - 1/\varepsilon_{r_1}\right)$$

$$\sigma_{P_{21}} = \vec{P}_2 \cdot (-\check{r}) = -\frac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_2}\right)\Big|_{R_2} = -\sigma_L \frac{R_1}{R_2}\left(1 - 1/\varepsilon_{r_2}\right)$$

$$\sigma_{P_2} = \vec{P} \cdot (+\check{r}) = \frac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_2}\right)\Big|_{R_3} = \frac{\sigma_{L_1} R_1}{R_3}\left(1 - 1/\varepsilon_{r_2}\right)$$

$$\vec{E} = \begin{cases} 0\ \check{r} & \text{si } r < R_1 \\ \dfrac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_1} r}\ \check{r} & \text{si } R_1 < r < R_2 \\ \dfrac{\sigma_{L_1} R_1}{\varepsilon_0 \varepsilon_{r_2} r}\ \check{r} & \text{si } R_2 < r < R_3 \\ 0\ \check{r} & \text{si } r > R_3 \end{cases}$$

$$\vec{D} = \begin{cases} 0\ \check{r} & \text{si } r < R_1 \\ \dfrac{\sigma_{L_1} R_1}{r} & \text{si } R_1 < r < R_3 \\ 0 & \text{si } r > R_3 \end{cases}$$

$$\vec{P} = \begin{cases} 0\ \check{r} & r < R_1 \\ \dfrac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_1}\right) \check{r} & R_1 < r < R_2 \\ \dfrac{\sigma_{L_1} R_1}{r}\left(1 - 1/\varepsilon_{r_2}\right) \check{r} & R_2 < r < R_3 \\ 0\ \check{r} & r > R_3 \end{cases}$$

Dielectricos: Ley de gauss: $\iint_{\Sigma} \vec{D} \cdot d\vec{s} = Q_L]_{\Sigma}$

$[\vec{D}] = C/m^2$ • $\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ ⇝ Vector desplazamiento electrico

$[\vec{P}] = C/m^2$ • $\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \varepsilon_0 \chi \vec{E}$ ⇝ Vector polarizacion $\vec{P} = \vec{0}$ si no hay dielectrico

$[\vec{E}] = N/C = V/m$ • $\varepsilon_r = 1 + \chi$ ⇝ χ: susceptibilidad electrica, ε_r: permiabilidad relativa $[\varepsilon_r] = 1$

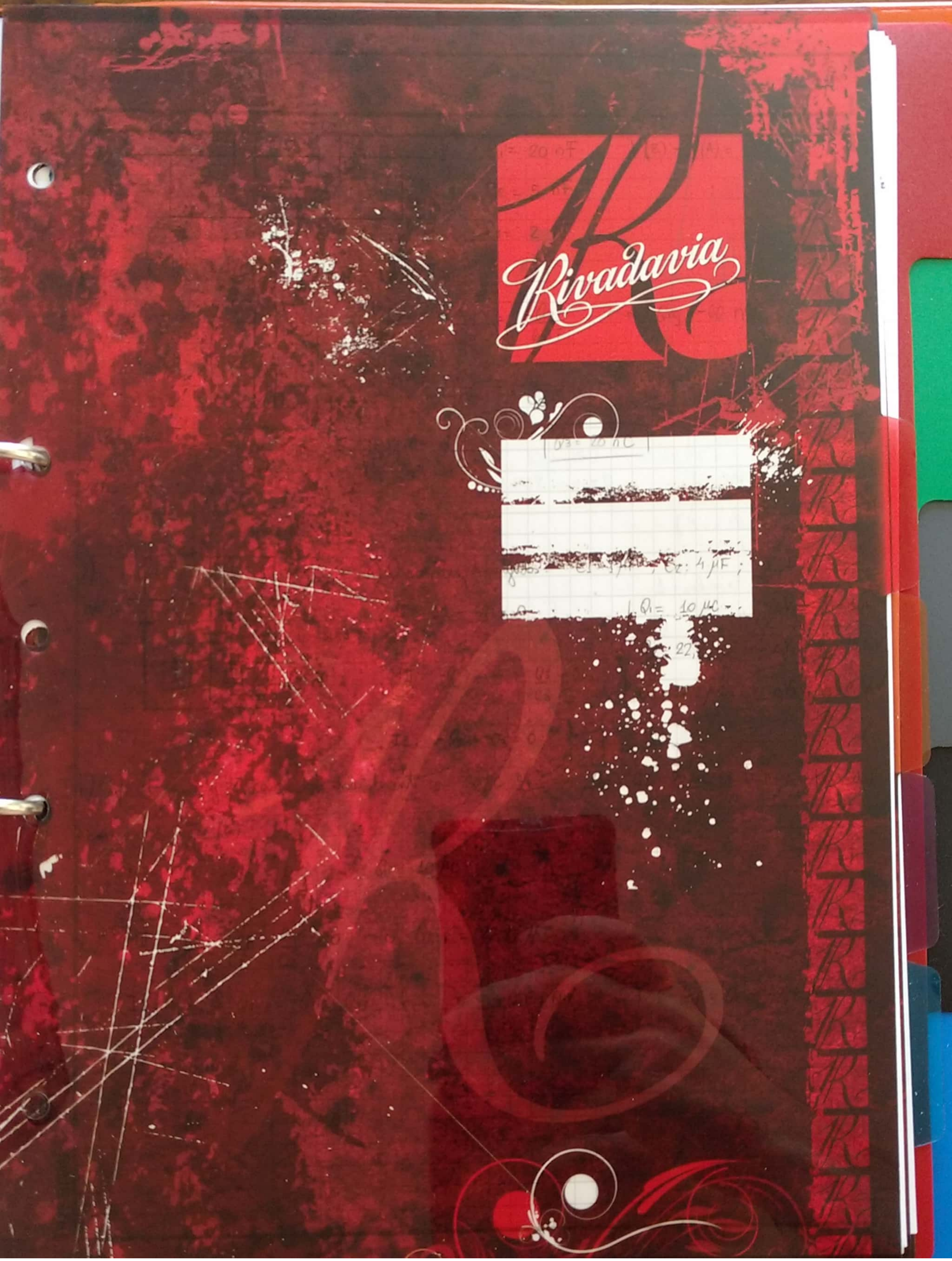
Hipotesis: material homogeneo, isotropo y lineal ⇒ $\vec{P}; \vec{D}; \vec{E}$ son colineales

$$Q_P = 0 = \int_S \sigma_P \, ds + \int_{Vol} \rho_P \, dVol \quad \begin{cases} \sigma_P = \vec{P} \cdot \check{\eta} & , \ \check{\eta} \text{ saliente} \rightsquigarrow \text{Densidad superficial de cargas polarizadas.} \\ \rho_P = -\bar{\nabla} \cdot \vec{P} & \rightsquigarrow \text{Densidad volumetrica de cargas polarizadas} \end{cases}$$

Si se tiene un dielectrico cargado (con carga libre) ⇒ Va a tener σ_P y ρ_P

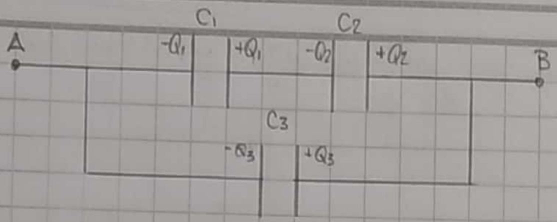
Relaciones • $\bar{\nabla} \cdot \vec{E} = \dfrac{\rho_L + \rho_P}{\varepsilon_0}$ • $\bar{\nabla} \cdot \vec{D} = \rho_L$ • $\nabla^2 V = \dfrac{-\rho_T}{\varepsilon_0}$

Condiciones de frontera: • $\vec{E}_{T_1} = \vec{E}_{T_2}$ • $D_{N1} - D_{N2} = \sigma_L$, σ_L: carga libre en la interfase.


Rivadavia

$C = \frac{Q}{\Delta V_p}$

1)

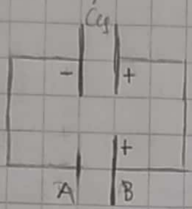


$C_1 = 20\ nF$ $\qquad V(B) - V(A) = 10V$

$C_2 = 5\ nF$

$C_3 = 2\ nF$

$$C_{eq} = \left(1/C_1 + 1/C_2 \right)^{-1} + C_3 = 6\ nF$$



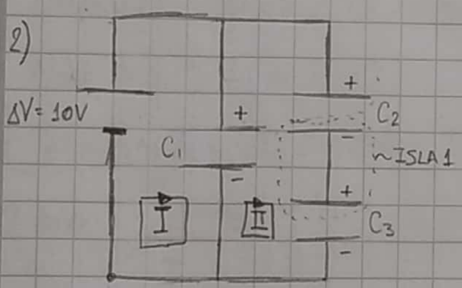
$$10V = -\frac{Q_{eq}}{C_{eq}} = \frac{-Q_{eq}}{6nF} \rightsquigarrow Q_{eq} = -60\ nC$$

$$10V = \frac{-Q_1}{C_1} + \frac{Q_2}{C_2}, \quad Q_1 = Q_2 = Q_{12} \Rightarrow \boxed{Q_{12} = 40\ nC}$$

$$10\ V = \frac{Q_3}{C_3} \qquad \boxed{Q_3 = 20\ nC}$$

$\Delta V_{P_1} = 2V$
$\Delta V_{P_2} = 8V$
$\Delta V_{P_3} = 10V$

2)



a) Capacitores descargados: $\quad C_1 : 1\mu F$; $C_2 : 4\mu F$; $C_3 : 5\mu F$

- MI: $10 - \frac{Q_1}{C_1} = 0$
- MII: $\frac{Q_1}{C_1} - \frac{Q_2}{C_2} - \frac{Q_3}{C_3} = 0$
- I1: $-Q_2 + Q_3 = 0$

$Q_1 = 10\ \mu C$, $\quad \Delta V_1 = \frac{Q_1}{C_1} = 10V$

$Q_2 = 22{,}2\ \mu C$, $\quad \Delta V_2 = \frac{Q_2}{C_2} = 5{,}6\ V$

$Q_3 = 22{,}2\ \mu C$, $\quad \Delta V_3 = \frac{Q_3}{C_3} = 4{,}44\ V$

b) Capacitores 2 y 3 cargados inicialmente con $20\ \mu C$ c/u $\Rightarrow$ $Q_{30} = Q_{20} = 20\ \mu C$ con la misma polaridad usada en el punto (a).

MI: $10 - \frac{Q_{1F}}{C_1} = 0 \rightsquigarrow Q_{1F} = 10\ \mu C$

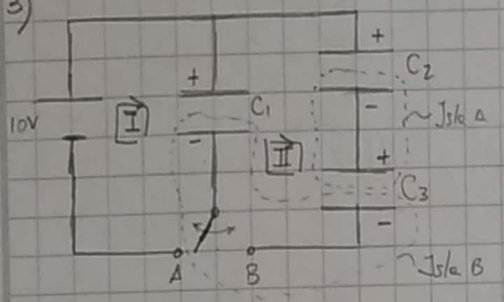
MII: $\frac{Q_{1F}}{C_1} - \frac{Q_{2F}}{C_2} - \frac{Q_{3F}}{C_3} = 0 \rightsquigarrow Q_{23F} = 22{,}22\ \mu C$

I1: $-Q_{2_0} + Q_{3_0} = -Q_{2F} + Q_{3f} \rightsquigarrow 0 = -Q_{2f} + Q_{3f} \rightsquigarrow Q_{3f} = Q_{2f} = Q_F = 42{,}2\ \mu C$

$\quad -20 + 20 = 0$

húsares

3)



a) La llave se conecta en A. $Q_{i0} = 0$

Malla I: $10 - \frac{Q_{1F}}{C_1} = 0$ $\boxed{Q_{1F} = 200\,\mu C}$

$U_1 = \frac{Q\,\Delta V}{2} = \frac{200 \cdot 10}{2} = 1000\,\mu J$

b) La llave se mueve a B: $Q_{10} = 200\,\mu C$ $Q_{20} = Q_{30} = 0$.

Malla II: $\Delta V_{P_1} - \Delta V_{P_2} - \Delta V_{P_3} = 0 \rightsquigarrow \frac{Q_{1F}}{C_1} - \frac{Q_{2F}}{C_2} - \frac{Q_{3F}}{C_3} = 0$

$\boxed{Q_{23} = 28{,}57\,\mu C}$

$\boxed{Q_{1F} = 171{,}43\,\mu C}$

Isla A: $-\cancel{Q_{20}}^{\,0} + \cancel{Q_{30}}^{\,0} = -Q_{2F} + Q_{3F} \rightsquigarrow Q_{2F} = Q_{3F} = Q_{23}$

Isla B: $-Q_{10} - \cancel{Q_{30}}^{\,0} = -Q_{1F} - Q_{3F} \rightsquigarrow -200 = -Q_{1F} - Q_{3F} \rightsquigarrow Q_{1F} = 200 - Q_{23}$

$U_1 = \frac{1}{2} \cdot \frac{Q_{1F}^2}{C_1} = 735\,\mu J$

$U_2 = \frac{1}{2} \frac{Q_{2F}^2}{C_2} = 40{,}81\,\mu J$ $U_{TOT} = 857{,}1\,\mu J < U(a)$

$U_3 = \frac{1}{2} \frac{Q_{3F}^2}{C_3} = 81{,}62\,\mu J$

c) $\varepsilon_{r_2} = 2$ $\varepsilon_{r_1} = \varepsilon_{r_3} = 1$

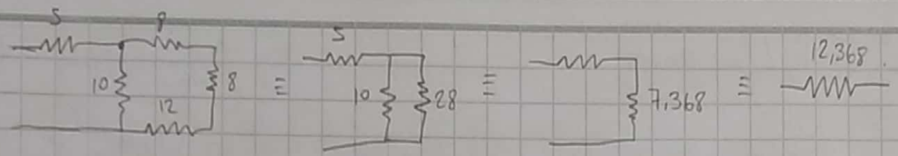
$C_{2_2} = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon_0 A}{d} \cdot \varepsilon_r = C_{2_1} \cdot \varepsilon_r = 20\,\mu F$

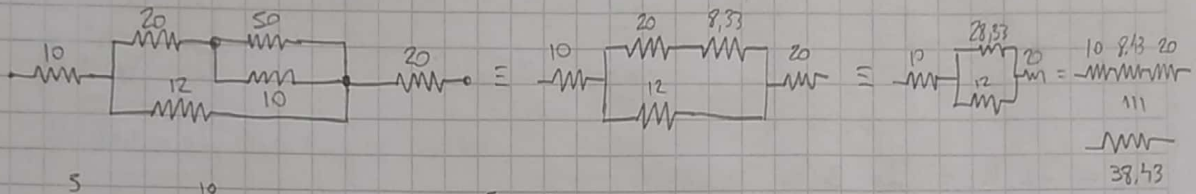
Con las mismas ecuaciones que (b) resulta que:

$\begin{cases} Q_{2F} = Q_{3F} = 33{,}33\,\mu C \\ Q_{1F} = 166{,}67\,\mu C \end{cases}$

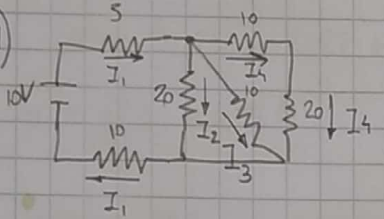
$U_1 = 694{,}5\,\mu J$
$U_2 = 27{,}8\,\mu J$ $\rightsquigarrow U_T = 833{,}33\,\mu J.$
$U_3 = 111{,}1\,\mu J$

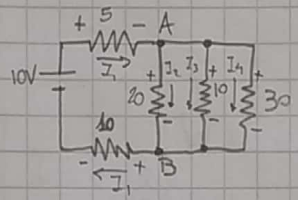
6)

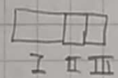




9)







Malla (I) $10 - I_1 . 5 - 20 I_2 - 10 I_1 = 0 \rightsquigarrow 10 - 15 I_1 - 20 I_2 = 0$ -

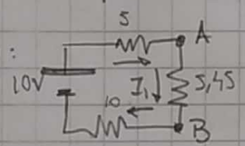
Malla (III) $-30 I_4 + 10 I_3 = 0$

Malla (II) $20 I_2 - 10 I_3 = 0$

Nodo A : $I_1 = I_2 + I_3$

$[I_1 = 0{,}46\,A ;\ I_2 = 0{,}15\,A ;\ I_3 = 0{,}31\,A ;\ I_4 = 0{,}1\,A]$

O bien:



$10\,V = 20{,}45\, I_1 \qquad I_1 = 0{,}49\,A$

$V_A - V_B = I_1\, 5{,}45 = 2{,}66\,V$

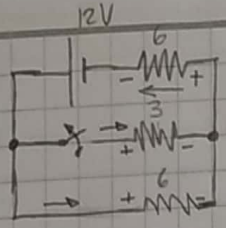
Volviendo al primer sist:

$V_A - V_B = I_3 . 10 \rightsquigarrow I_3 = 0{,}266\,A$

$V_A - V_B = I_4 . 30 \rightsquigarrow I_4 = 0{,}089\,A$

$V_A - V_B = I_2 . 20 \rightsquigarrow I_2 = 0{,}133\,A$

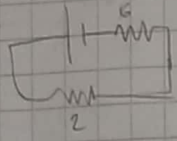
10)



a) Llave abierta $-I\,6 - I\,6 + 12 = 0$ $I = 1A$.

$Pot_{ent} = I\;e_m = 12W$

b) Llave cerrada



$12 = 8 I_1$ $\boxed{I_1 = 1{,}5A}$ $I_1 = I_2 + I_3$ $I_1 = 4 I_3$

$-6 I_3 + 3 I_2 = 0$ $\boxed{I_3 = 0{,}375A}$

$-3I_3 + I_2 = 0$ $\boxed{I_2 = 1{,}125A}$

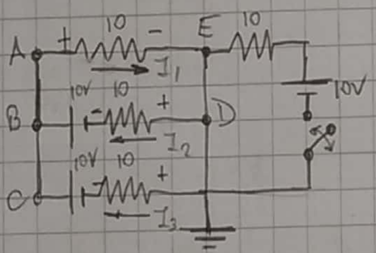
$\Delta V_2 = I_2 . R_2 = I_2 . 3 = 3{,}375V$

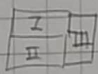
$Pot_{dis} = \Delta V_2 . I_2 = 3{,}8W$

c) $Pot_{ent} = I_1 . e_m = 1{,}5A . 12V = 18W$

d) Consumo (dos días = 48hs) = 18 48hs = 0,864 KWh

11)





Malla I: $-I_1\,\cancel{10} - \cancel{10}\,I_2 + \cancel{10}^{1} = 0$. $I_{23} = 1/3\,A$

Malla II: $-10 + 10 . I_2 - 10 . I_3 + 10 = 0 \rightarrow I_2 = I_3 = I_{23}$

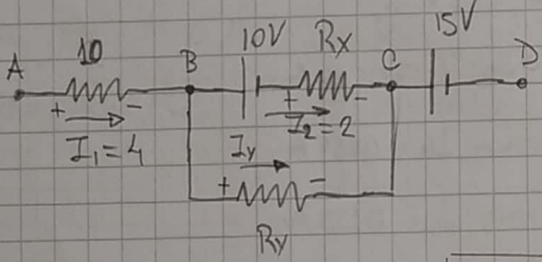
Nodo B: $I_2 + I_3 = I_1 = 2I_{23} = 2/3\,A$

$\Delta V_1 = 6{,}67V$
$\Delta V_2 = 3{,}33V$
$\Delta V_3 = 3{,}33V$

$V_A - V_T = 6{,}67V$
$V_B - V_T = 6{,}67V$
$V_C - V_T = 6{,}67V$

} No depende de la llave L.

12)



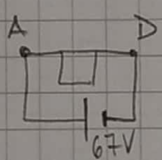
$V_B - V_C = 12V$

Dat: $V_B - V_C = R_y I_y = 12$ $\boxed{R_y = 6\,\Omega}$

N_B: $I_1 = I_2 + I_y$ $\boxed{I_y = 2A}$

M_B: $-I_y . R_y + R_x I_2 + 10V = 0$ $\boxed{R_x = 1\,\Omega}$

c) $V_A - V_D = 15 + I_y R_y + 10 I_1 = 67V$

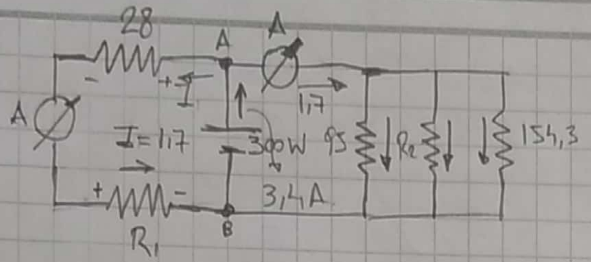


d) $Pot_{ent} = I_1 . 67V = 268W$

húsares

Scanned by CamScanner

13)



$R_1 = ?$

$R_2 = ?$

$Pot_{ent} = I\, e_m = 3{,}4\, e_m = 300\, W \qquad e_m = 88{,}2\, V$

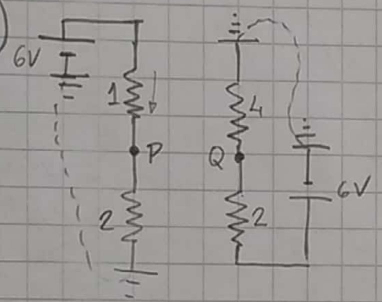
$V_A - V_B = 88{,}2\,V = +I\, 28 + I \cdot R_1 \rightarrow \boxed{R_1 = 23{,}88\ \Omega}$

$I_{95} = \dfrac{88{,}2}{95} = 0{,}93\, A$

$I_{154,3} = \dfrac{88{,}2}{154{,}3} = 0{,}57\, A$

$I_{R_2} = 0{,}2\, A. \qquad 88{,}2 = I_{R_2} \cdot R_2 \Rightarrow \boxed{R_2 = 441\ \Omega}$

14)



$V_P - V_Q = ?$

Malla I : $6V - 3 I_1 = 0 \rightarrow I_1 = 2\, A$

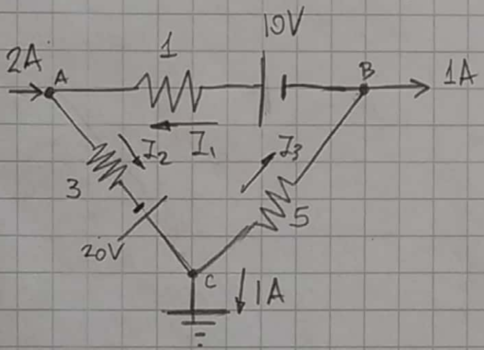
Malla II : $6V - 6 I_2 = 0 \rightarrow I_2 = 1\, A$

$V_P - V_T = 6 - 1 \cdot 2 = 4V$

$V_Q - V_T = 6 - 2 \cdot 1 = 4V$

$\boxed{V_P - V_Q = 0V}$

15)



Nodo C: $I_2 = 1 + I_3$.

Nodo B: $I_3 = I_1 + 1$

Malla (B): $I_3 \cdot 5 - 20 + 3 I_2 + I_1\, 1 - 10 = 0$

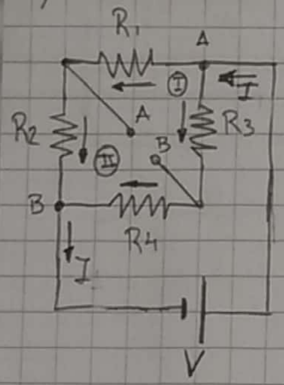
$I_1 = 2{,}11\, A \qquad I_2 = 4{,}11\, A \qquad I_3 = 3{,}11\, A$

$V_B - V_A = I_1 \cdot 1 - 10 = -7{,}89\, V$

$V_C - V_A = -3 I_2 + 20 = 7{,}67\, V$

$V_B - V_C = -5 I_3 = -15{,}55\, V$

16) Puente de Wheanstone



$$I = I_1 + I_3 = I_2 + I_4$$

$$\overset{\frown}{M I_A}: \ -I_3 R_3 + I_1 R_1 = 0 \ \rightarrow \ I_1 R_1 = I_3 R_3 \ \rightarrow \ I_3 = \frac{I_1 R_1}{R_3}$$

$$\overset{\frown}{M II_B} = \ R_2 I_2 - I_4 R_4 = 0 \ \rightarrow \ I_2 R_2 = I_4 R_4 \ \rightarrow \ I_4 = \frac{I_2 R_2}{R_4}$$

$$I_1 \left(1 + \frac{R_1}{R_3}\right) = I_2 \left(1 + \frac{R_2}{R_4}\right)$$

$$I_1 = I_2 \iff \boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$$

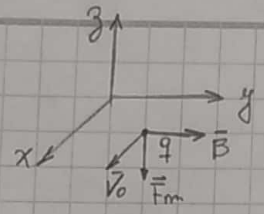
Rivadavia

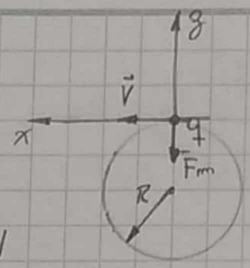
guía 4

1) $\vec{V_0} = 10^5\ m/seg\ \check{i}$

$\vec{B} = 0{,}4T\ \check{j}$

$q = -1{,}6.10^{-19}\ C$





a) $\vec{F_m} = q\bar{V} \times \bar{B} = q\,VB\ \check{k} = -6{,}4.10^{-15}\ \check{k}\ N$

$\vec{F_m} = m.\vec{a} \Rightarrow F_m = m.a_c = m.\frac{V^2}{R}$, $m = 9{,}11.10^{-31}\ Kg \rightsquigarrow R = \frac{mV^2}{F_m} = 1{,}42.10^{-6}\ m$

b) Mov circular uniforme (m.c.u):

$\vec{r} = (R\cos(wt); 0; R\sin(wt) - R)$, $w = \sqrt{\frac{a_c}{R}} = \frac{V}{R} = 7{,}02.10^{10}\ 1/seg$

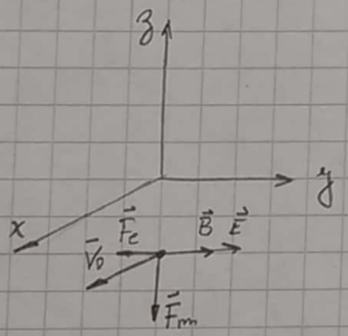
c) La energía cinética del electrón se conserva ya que $\bar{F_m} \perp \vec{V}$

2) $\vec{E} = 10.000\ \frac{V}{m}\ \check{j}$

$\vec{B} = 0{,}4\ T\ \check{j}$

$q = -1{,}6.10^{-19}\ C$

$\vec{V_0} = 10^5\ m/seg\ \check{i}$



$\vec{F_q} = q\vec{E} + q\vec{V} \times \bar{B}$

$\vec{F_q} = (0, qE, 0) + (0, 0, qVB) = (0, qE; qVB) = (0; -1.6.10^{-15}; -6{,}4\,10^{-15})\ N$

La energía cinética varía con el tiempo ya que la fuerza electrostática acelera la partícula

4) $F_{m_1} = -qVB = m\frac{V^2}{R_1} = m\frac{V^2}{R}$ $\qquad q_1 = -\frac{mV}{BR}$ $\qquad q_2/q_1 = 1/2$

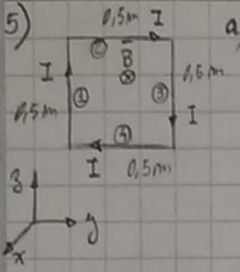
$q_3/q_1 = 0$

$F_{m_2} = -qVB = m\frac{V^2}{R_2} = m\frac{V^2}{2R}$ $\qquad q_2 = -\frac{mV}{2BR}$ $\qquad q_4/q_1 = -2/3$

$F_{m_3} = -qVB = m\frac{V^2}{R_3} = 0$ $\qquad q_3 = 0$ $\qquad q_5/q_1 = -2$

$F_{m_4} = qVB = \frac{mV^2}{R_4} = \frac{2mV^2}{3R}$ $\qquad q_4 = \frac{2mV}{3BR}$

$F_{m_5} = qVB = \frac{mV^2}{R_5} = \frac{2mV^2}{R}$ $\qquad q_5 = \frac{2mV}{BR}$

5) 

a) $\vec{F}_m = \int_L I\, d\vec{l} \times \vec{B}$ $\begin{cases} I = 5\,A \\ B = 0{,}3\,T \end{cases}$

$\vec{F}_{m_1} = -I\,L\,B\,\check{j} = -0{,}75\,N\,\check{j}$ $\quad F_{m_3} = I\,L\,B\,\check{k} = 0{,}75\,N\,\check{k}$

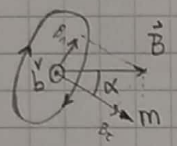
$\vec{F}_{m_2} = I\,L\,B\,\check{j} = 0{,}75\,N\,\check{j}$ $\quad F_{m_4} = -I\,L\,B\,\check{k} = -0{,}75\,N\,\check{k}$

$\vec{F}_R = (0,0,0)$

$[T] = \frac{N\,s}{m\,C}$

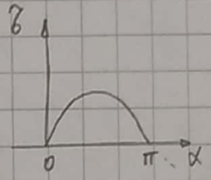
b) Momento magnético: $\vec{\mu}_m = I\,\vec{S} = 5\,A \cdot (0{,}5\,m)^2(\check{-i}) = 1{,}25\,A\,m^2\,(\check{-i})$ $\quad$ ijkij

Torque: $\vec{\tau} = \underset{i}{\vec{\mu}_m} \times \underset{k}{\vec{B}} = -\mu_m B\,(-\check{j}) = 1{,}25\,A\,m^2 \cdot 0{,}3\,T\,\check{j} = 0{,}375\,T\,A\,m^2\,\check{j}$

7) 

a) $\vec{\mu}_m = NIS = I\,\pi R^2 = 0{,}377\,A\,m^2\,\check{m}$

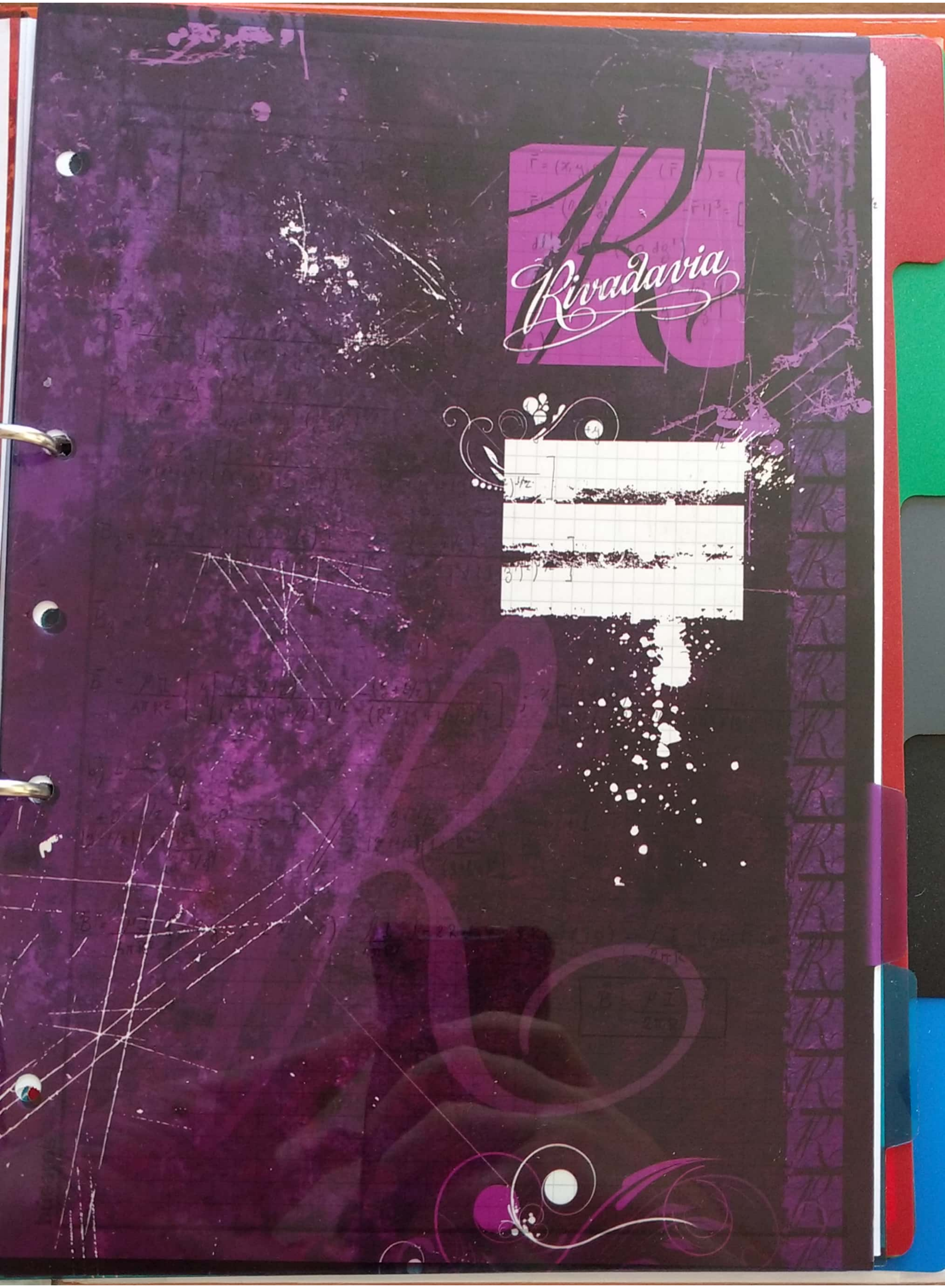
b) $\vec{\tau} = \vec{\mu}_m \times \vec{B} = \mu_m B \operatorname{sen}\alpha\,\check{b}$



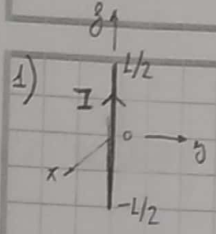
c) $N = 50$ espiras

a') $\vec{\mu}_m = NIS = 18{,}85\,A\,m^2$

b') $\vec{\tau} = \vec{\mu}_m \times \vec{B} = \mu_m B \operatorname{sen}\alpha\,\check{b}$


Rivadavia

1)



$$\vec{B} = \frac{\mu_0}{4\pi}\int_L \frac{I\,d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$\vec{r} = (x, y, z)$ $\quad (\vec{r}-\vec{r}') = (x, y, z-z')$

$\vec{r}' = (0, 0, z')$ $\quad |\vec{r}-\vec{r}'|^3 = \left[x^2+y^2+(z-z')^2\right]^{3/2}$

$d\vec{l}' = d\vec{r}' = (0, 0, dz')$

$$d\vec{l} \times (\vec{r}-\vec{r}') = \begin{vmatrix} 0 & 0 & dz' \\ x & y & z-z' \end{vmatrix} = (-y\,dz' ; +x\,dz' ; 0)$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{-L/2}^{L/2} \frac{(-y\,dz' ; x\,dz', 0)}{(x^2+y^2+(z-z')^2)^{3/2}} = (B_x ; B_y, B_z)$$

$$B_x = \frac{\mu_0 I y}{4\pi}\int_{-L/2}^{L/2}\frac{-dz}{(x^2+y^2+(z-z')^2)^{3/2}} = \frac{\mu_0 I y}{4\pi}\cdot\frac{1}{(x^2+y^2)}\,\frac{(z-z')}{(x^2+y^2+(z-z')^2)^{1/2}}\Bigg|_{-L/2}^{L/2}$$

$u = z - z'$

$du = -dz'$

$$= \frac{\mu_0 I y}{4\pi(x^2+y^2)}\cdot\left[\frac{(z-L/2)}{(x^2+y^2+(z-L/2)^2)^{1/2}} - \frac{(z+L/2)}{(x^2+y^2+(z+L/2)^2)^{1/2}}\right]$$

$$B_y = \frac{\mu_0 I x}{4\pi(x^2+y^2)}\cdot\left[\frac{(z+L/2)}{(x^2+y^2+(z+L/2)^2)^{1/2}} - \frac{(z-L/2)}{(x^2+y^2+(z-z')^2)^{1/2}}\right]$$

$$\vec{B}_z = 0$$

$-1 \quad - \quad (+1) \qquad\qquad 1 \quad - \quad (-1)$

$$\vec{B} = \frac{\mu I}{4\pi R^2}\left\{ y\left[\frac{(z-L/2)}{(R^2+(z-L/2)^2)^{1/2}} - \frac{(z+L/2)}{(R^2+(z+L/2)^2)^{1/2}}\right] ; x\left[\frac{(z+L/2)}{(R^2+(z+z')^2)^{1/2}} - \frac{(z-L/2)}{(R^2+(z-z')^2)^{1/2}}\right] ; 0\right\}$$

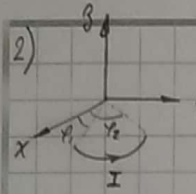
b) $L \to \infty$

$$\frac{\overbrace{z-L/2}^{-1}}{|z-L/2|\left(1+\underbrace{\frac{R^2}{(z-L/2)^2}}_{0}\right)} \to -1 \qquad // \qquad \frac{z+L/2}{|z+L/2|\left(1+\frac{R^2}{(z+L/2)^2}\right)} \to +1$$

$$\vec{B} = \frac{\mu I}{4\pi R^2}(-2y ; 2x ; 0) = \frac{\mu I}{4\pi R^2}(-2R\sin\varphi ; 2R\cos\varphi ; 0) = \frac{\mu I}{2\pi R}(-\sin\varphi ; \cos\varphi ; 0)$$

$$\boxed{\vec{B} = \frac{\mu I}{2\pi R}\,\check{\varphi}}$$

2)



$\vec{r} = (0,0,z)$

$\vec{r}\,' = (R\cos\varphi';\ R\,\text{sen}\,\varphi';\ 0)$

$(\vec{r}-\vec{r}\,') = (-R\cos\varphi';\ -R\,\text{sen}\,\varphi';\ z)$

$|\vec{r}-\vec{r}\,'|^3 = (R^2+z^2)^{3/2}$

$d\vec{r}\,' = (-R\,\text{sen}\,\varphi'\,d\varphi';\ R\cos\varphi'\,d\varphi';\ 0)$

$$d\vec{r}\,' \times (\vec{r}-\vec{r}\,') = \begin{vmatrix} -R\,\text{sen}\,\varphi'\,d\varphi' & R\cos\varphi'\,d\varphi' & 0 \\ -R\cos\varphi' & -R\,\text{sen}\,\varphi' & z \end{vmatrix} = (zR\cos\varphi'\,d\varphi';\ zR\,\text{sen}\,\varphi'\,d\varphi';\ R^2\,d\varphi')$$

Biot y Savart:

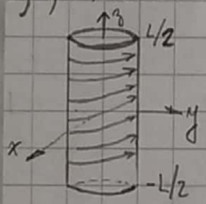
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{\varphi_1}^{\varphi_2} \frac{(zR\cos\varphi'\,d\varphi';\ zR\,\text{sen}\,\varphi'\,d\varphi';\ R^2\,d\varphi')}{(R^2+z^2)^{3/2}}$$

$$\left[\ \vec{B} = \frac{\mu_0 I}{4\pi(R^2+z^2)^{3/2}}\Big(zR(\text{sen}\,\varphi_2-\text{sen}\,\varphi_1);\ zR(\cos\varphi_1-\cos\varphi_2);\ R^2(\varphi_2-\varphi_1)\Big)\ \right]$$

b) $\varphi_1 = 0 \ \wedge\ \varphi_2 = 2\pi$

$$\boxed{\vec{B} = \left(0,0;\ \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\right)}$$

3) a) Solenoide corto:

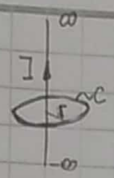


$\vec{B} = B_z\,\check{k}$

$N - L$

$x - dz \ \rightsquigarrow\ x = \frac{N}{L}\,dz$

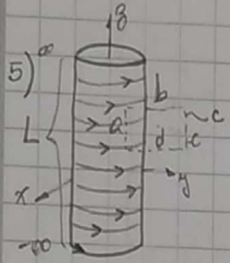
$$dB_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}} \cdot \frac{N}{L}\,dz$$

4) 

Ley de Ampere: $\oint_C \vec{B}\cdot\vec{dl} = I_{conc}\cdot\mu_0$

$$\begin{cases} \vec{B} = B(r)\,\check{\varphi} \\ \vec{dl} = r\,d\varphi\,\check{\varphi} \end{cases} \qquad \int_0^{2\pi} B(r)\cdot d\varphi = I\mu_0$$

$$B\,2\pi r = I\mu_0 \rightsquigarrow \boxed{\vec{B}(r) = \frac{I\mu_0}{2\pi r}\,\check{\varphi}}$$

5) 

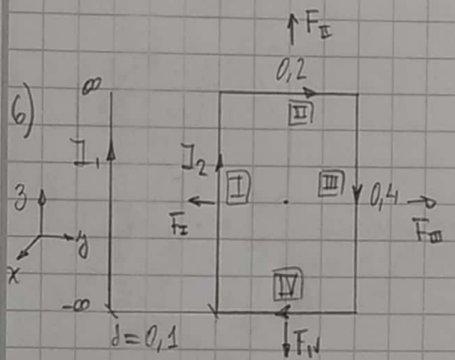
Se tiene un solenoide infinito $\Rightarrow$ $\begin{cases} \vec{B} = B_z\,\check{k} \\ \vec{dl} = dz\,\check{k} \end{cases}$ $\vec{B}$: uniforme en cualquier punto del eje z

$B \perp dl$ $\;b\text{-}c$

Ley de Ampere: $\int_C \vec{B}\cdot\vec{dl} = I_{conc}\,\mu_0 = I\,N.\frac{L_a}{L}\,\mu_0 = \int_a B\,dz + \int_b^{0} + \int_c^{0} + \int_d$

$$B\,\cancel{L_a} = \mu_0\,I\,\frac{\cancel{L_a}}{L}\,N \rightsquigarrow \boxed{\vec{B} = \frac{\mu_0 I N}{L}\,\check{k} = \mu_0\,I\,m\,\check{k}}$$

b)

6) 

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\,\check{\varphi}$$

La espira esta inmersa en $\vec{B} = \frac{\mu_0 I_1}{2\pi d_i}(-\check{x})$

Tramo I: $*$ $\vec{F}_I = \int_I I_2\,\vec{dl}\times\vec{B}$, $\vec{dl}\times\vec{B} = \begin{vmatrix} 0 & 0 & dz \\ -B_x & 0 & 0 \end{vmatrix} = (0; -B\,dz, 0)$

$$\vec{F}_I = -I_2 B\,0{,}4\,\check{j} = -\frac{I_1 I_2 \mu_0\,0{,}4}{2\pi\,0{,}1}\,\check{j} = -8.10^{-7}\,N\,\check{j}$$

$*$ $\vec{F}_{III} = \frac{I_1 I_2 \mu_0\,0{,}4}{2\pi\,0{,}3} = 2{,}67.10^{-7}\,N\,\check{j}$

$*$ $\vec{F}_{II} = \int_{II} I_2\,\vec{dl}\times\vec{B}$, $\vec{dl}\times\vec{B} = \begin{vmatrix} 0 & dy & 0 \\ -B_x & 0 & 0 \end{vmatrix} = (0, 0, B_x\,dy)$

$$\vec{F}_{II} = I_2\int_{II} \frac{I_1\mu_0}{2\pi y}\,dy = \frac{I_2 I_1 \mu_0}{2\pi}\ln\left(\frac{0{,}3}{0{,}1}\right) = 2{,}2.10^{-7}\,N\,\check{k}$$

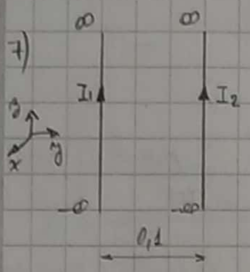
$*$ $\vec{F}_{IV} = -2{,}2.10^{-7}\,N\,\check{k}$

b) ~~$\vec{\mu}_m = I\,\vec{S} = I_2\, 0{,}4 \cdot 0{,}2 = 0{,}008 \quad \check{n} = 0{,}008\, \check{i}$~~

~~$\vec{\tau} = \vec{\mu}_m \times \vec{B} = \vec{0} \qquad \vec{\mu}_m \parallel \vec{B}$~~

$\vec{M} = \vec{F} \times \vec{r} = \vec{0} \quad \forall\, i,\ i = \mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}$ ya que $\vec{r} \parallel \vec{F}$

7)



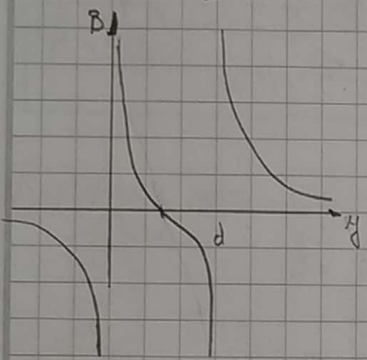
$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\,\check{\varphi} \quad , \quad \vec{B} = -\frac{\mu_0 I_1}{2\pi y}\,\check{x}$$

$$\vec{F}_{21} = I_2 \int \vec{dl} \times \vec{B}_1 = \begin{vmatrix} 0 & 0 & dz \\ -B_x & 0 & 0 \end{vmatrix} I_2 = -I_2 \int_0^L B\, dz = -I_2 B L\, \check{j}$$

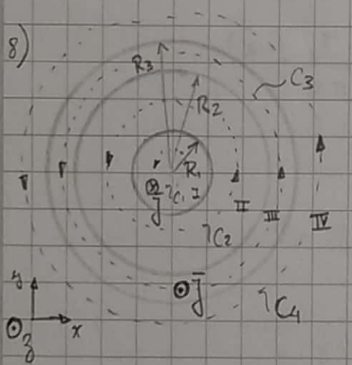
$$\frac{\vec{F}_{21}}{L} = -\frac{\mu_0 I_1 I_2}{2\pi (0{,}1)} = -8 \cdot 10^{-4} \frac{N}{m}\,\check{j}$$

Si I_2 circulara hacia el otro lado,

$$\vec{F}_{21} = -I_2 \int_L^0 B\, dz = -I_2 B(-L)\,\check{j} = I_2 B L\,\check{j} \Rightarrow \frac{\vec{F}_{21}}{L} = 8\, 10^{-4} \frac{N}{m}\,\check{j}$$



8)



Cilindro 1: densidad de corriente volumetrica : $\vec{J} = -J\,\check{k}$

Cilindro 2: " " " " : $\vec{J} = J\,\check{k}$

$\boxed{\mathrm{I}}$ $$\int_{C_1} B\, dl = I_{conc} \cdot \mu_0 = J \cdot \pi r^2 \mu_0 = B\, 2\pi r$$

$$\vec{B} = -\frac{J r \mu_0}{2}\,\check{\varphi} \quad , \quad I = \iint_S \vec{J} \cdot \vec{ds} = J\, \pi R_1^2$$

$$\vec{B} = -\frac{I_1 r \mu_0}{2\pi R_1^2}\,\check{\varphi} \quad \text{si } r < R_1$$

$\boxed{\mathrm{II}}$ $$\int_{C_2} \vec{B} \cdot \vec{dl} = \mu_0 I_{conc} \rightsquigarrow B\, 2\pi r = -\mu_0 J \pi R_1^2$$

$$\vec{B} = -\frac{\mu_0 J R_1^2}{2r}\,\check{\varphi} = -\frac{\mu_0 I_1}{2\pi r}\,\check{\varphi} \quad \text{si } r \in (R_1, R_2)$$

$\boxed{\mathrm{III}}$ $$\int_{C_3} \vec{B} \cdot \vec{dl} = \mu_0 I_{conc} = \mu_0 \left(-J \pi R_1^2 + J \pi (r^2 - R_2^2)\right) = \mu_0 \left[J \pi (r^2 - R_1^2 - R_2^2) \right] = B\, 2\pi r$$

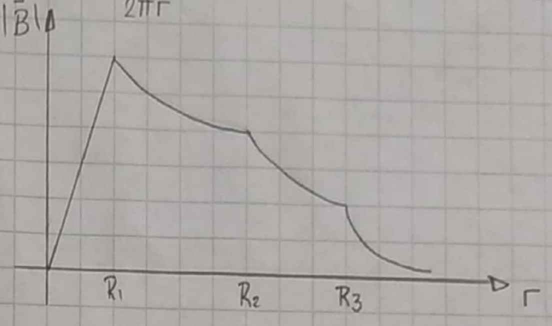
$$\vec{B} = \frac{\mu_0 J}{2r} (r^2 - R_1^2 - R_2^2)\,\check{\varphi} = -\frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_2}{2\pi r} \frac{(r^2 - R_2^2)}{(R_3^2 - R_2^2)}\,\check{\varphi} \quad , r \in (R_2, R_3)$$

$J = \frac{I}{\pi r^2}$

IV) $\int_{C_4} \vec{B}\cdot d\vec{l} = B\, 2\cancel{\pi} r = \mu_0 I_{conc} = \mu_0 J \cancel{\pi} \left(-R_1^2 + R_3^2 - R_2^2\right)$

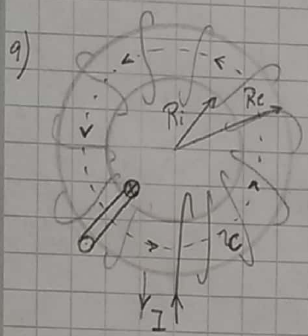
$$\vec{B} = \frac{\mu_0 \left(R_3^2 - R_2^2 - R_1^2\right) J}{2r}\hat{\varphi} = \frac{\mu_0}{2\pi r}(I_1 + I_2)\hat{\varphi} \qquad r > R_3$$

$$\vec{B} = \begin{cases} -\frac{J r \mu_0}{2}\hat{\varphi} & r < R_1 \\ -J\frac{\mu_0 R_1^2}{2r}\hat{\varphi} & R_1 < r < R_2 \\ J\frac{\mu_0}{2r}\left(r^2 - R_1^2 - R_2^2\right)\hat{\varphi} & R_2 < r < R_3 \\ J\frac{\mu_0}{2r}\left(R_3^2 - R_1^2 - R_2^2\right)\hat{\varphi} & R_3 < r \end{cases}$$

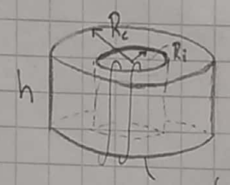


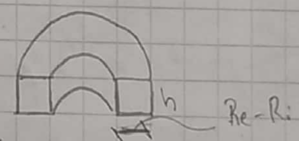
$B_{ext} = 0 = J\frac{\mu_0}{2r}\left(R_{3_x}^2 - R^2 - R_2^2\right) \Rightarrow R_{3x} = \sqrt{R_1^2 + R_2^2} \rightsquigarrow \boxed{R_3 = 2{,}7 \text{ cm}}$

9)



$N = 5000$





a) $\vec{B} = \begin{cases} \frac{\mu_0 N I}{2\pi r}\hat{\varphi} & r \in (R_i, R_e) \\ 0 & \text{Otro caso} \end{cases}$

$\int_c \vec{B}\cdot d\vec{l} = \mu_0 I_{conc} = \mu_0 N I$

b) $\vec{B} = \begin{cases} \frac{\mu_0 N I}{2\pi R_M}\hat{\varphi} & r \in (R_i ; R_e) \\ 0 & r > R_e \wedge r < R_i \end{cases}$

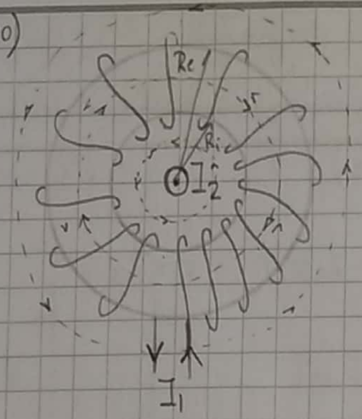
a) $\Phi_B(S) = \iint_S \vec{B}\cdot d\vec{s} = \int_0^h \int_{R_i}^{R_e} \frac{\mu_0 N I}{2\pi r}\, dr\, dz = \frac{\mu_0 N I}{2\pi} \ln\left(\frac{R_e}{R_i}\right) h = 6{,}13\cdot 10^{-7}\ \text{Wb}$

b) $\Phi_B(S) = \iint_S \vec{B}\cdot d\vec{s} = B \iint_S ds = B \quad R_M = \frac{\mu_0 N I}{2\pi} h = 4{,}8\, 10^{-7}\ \text{Wb}$

En este caso, no se puede usar el radio medio ya que es un Toroide grueso.

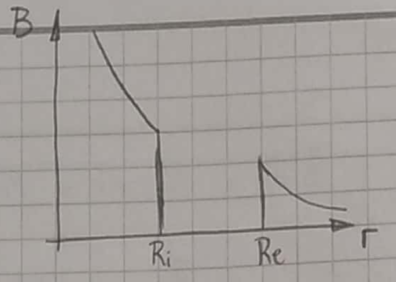
Scanned by CamScanner

10)



$$\vec{B}_T = \frac{\mu_0 N I_1}{2\pi r}\check{\varphi}$$

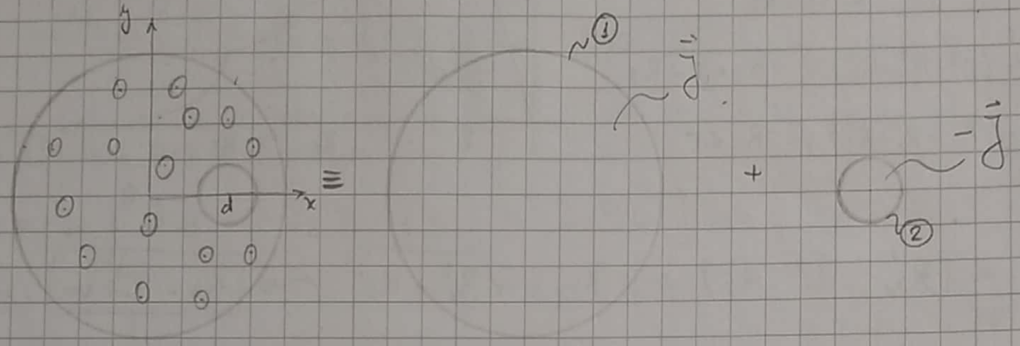
$$\vec{B}_I = \frac{\mu_0 I_2}{2\pi r}\check{\varphi}$$



$$\frac{\mu_0 N I_1}{2\pi r} = \frac{\mu_0 I_2}{2\pi r}$$

$$I_2 = N I_1$$

11)

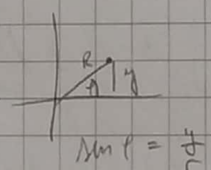


J uniforme $\quad |\vec{J}_1| = |\vec{J}_2|$

$$\vec{B}_1 = \frac{J r \mu_0}{2}\check{\varphi} = \frac{J\mu_0}{2} r(-\sin\varphi\,;\cos\varphi) = \frac{J\mu_0}{2} r\left(-\frac{y}{r}\,;\frac{x}{r}\right)$$

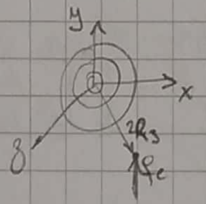
$$\vec{B}_2 = -\frac{J r' \mu_0}{2}\check{\varphi}' = -\frac{J\mu_0}{2} r'(-\sin\varphi'\,;\cos\varphi') = -\frac{J\mu_0}{2} r'\left(-\frac{y'}{r'}\,;\frac{x'}{r'}\right) = -\frac{J\mu_0}{2}(-y\,;x-d)$$

$$\vec{B} = \left(0\,;\frac{J\mu_0 d}{2}\,;0\right)$$

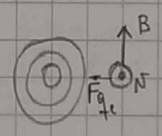


12) a)

$$\vec{B} = \begin{cases} \frac{I_1 r \mu_0}{2\pi R_1^2}\check{\varphi} & r < R_1 \\ \frac{I_1 \mu_0}{2\pi r}\check{\varphi} & r \in (R_1, R_2) \\ \frac{I_1\mu_0}{2\pi r} + \frac{I_2\mu_0}{2\pi r}\left(\frac{r^2 - R_2^2}{R_3^2 - R_2^2}\right)\check{\varphi} & r \in (R_2, R_3) \\ \frac{\mu_0}{2\pi r}(I_1 + I_2)\check{\varphi} & r > R_3 \end{cases}$$



b) $q_e = q \quad \wedge \quad \vec{V} = v\,\check{k}$



$$\vec{B}_\varphi = \frac{\mu_0}{2\pi(2R_3)}(I_1 + I_2)\check{\varphi}$$

$$\vec{F}_{q_e} = q_e\,\vec{v}\times\vec{B}_\varphi = q_e v B_\varphi(-\check{r}) \quad , \quad q_e < 0 \quad \Rightarrow \quad \vec{F}_{q_e} = |q| v B_\varphi \check{r}$$

c) $E_c = \frac{1}{2} m_e v^2 \rightsquigarrow$ No cambia ; $\vec{B} = \vec{0}$ en $r > R_3$ si $\boxed{I_1 = -I_2}$

MEDIOS MATERIALES

Ley de Ampere: $\oint_{\partial S} \vec{B} \cdot \vec{dl} = \mu_0 \left[I_{S} + I_{m} \right]_{conc}$

Ley de Gauss: $\oiint_{S} \vec{B} \cdot \vec{dS} = 0$

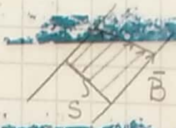
Ley de Ampere generalizada: $\oint_{\partial S} \vec{H} \cdot \vec{dl} = I_{F}\Big|_{conc} = FMM$

Tipos de circuitos magnéticos:

- Sección delgada ($\sqrt{S} \ll L_{m}$) : S: sección, L_m: long. media

$$\Phi_B(S) = \iint_S \vec{B} \cdot \vec{dS} = B \cdot S$$

$\vec{B}$ unif



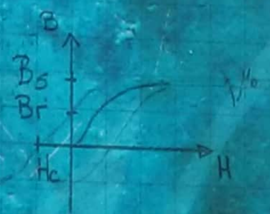
- Sección gruesa:

$$\Phi_B(S) = \iint_S \vec{B} \cdot \vec{dS}$$

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

- Lineales: $\vec{B} = \mu_0 \mu_r \vec{H}$

- No lineales: histéresis



B_S: campo inducción de saturación
B_r: " " remanencia
H_c: campo magnético coercitivo

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \frac{d\,\vec{\mu}_m}{d\,Vol}$$

Condiciones de frontera:

- $B_{1N} = B_{2N}$
- $H_{1T} = H_{2T}$

Unidades: $[B] = T$

$[H] = [M] = A/m$

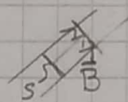
MEDIOS MATERIALES

Ley de Ampere: $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \cdot [I_V + I_m]_{conc}$ Ley de Gauss $\oint\!\!\oint_{\Sigma} \vec{B} \cdot d\vec{s} = 0$

Ley de Ampere generalizada: $\oint_C \vec{H} \cdot d\vec{l} = I_V \Big|_{conc} = FMM$

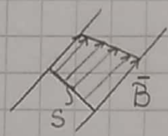
Tipos de circuitos magneticos:

- Seccion delgada $(\sqrt{S} \ll L_m)$ S: seccion L_m: long media



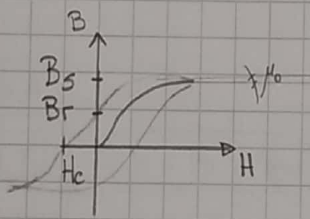
$$\Phi_B(S) = \iint_S \vec{B} \cdot d\vec{s} \underset{\substack{\uparrow \\ B\ unif}}{=} B\,S$$

- Seccion gruesa $\Phi_B(S) = \iint_S \vec{B} \cdot d\vec{s}$



△ Lineales $\vec{B} = \mu_0 \mu_r \vec{H}$ $\vec{M} = \dfrac{\vec{B}}{\mu_0} - \vec{H}$

△ No lineales: histeresis



B_s: campo induccion de saturacion
B_r: " " remanence
H_c: campo magnetico coercitivo.

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \frac{d\vec{\mu}_m}{d\,Vol}$$

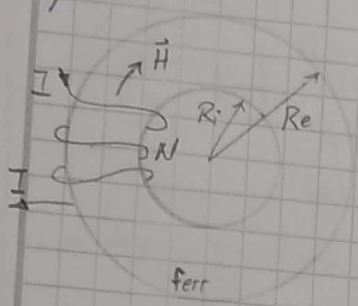
Condiciones de frontera: * $B_{1N} = B_{2N}$

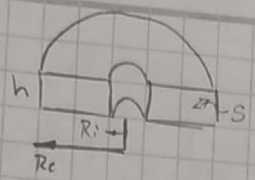
* $H_{1T} = H_{2T}$

Unidades: $[B] = T$

$[H] = [M] = A/m$

1)





Ley de Ampere generalizada:

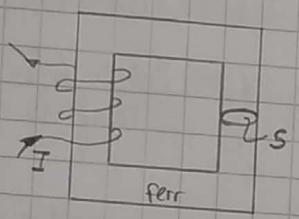
$$\oint_C \vec{H}\cdot d\vec{l} = I_c$$

- Secc delgada $\rightsquigarrow$ $H\, 2\pi r_i = NI$ $\qquad \vec{H} = \dfrac{NI}{2\pi r}\,\check{\varphi}$
- Mat lineal $\rightsquigarrow$ $\bar{B} = \dfrac{\mu_0 \mu_r NI}{2\pi r}\,\check{\varphi}$

b) $\Phi_B(S) = \iint_S \bar{B}\cdot d\bar{s} = \dfrac{h\,\mu_0\mu_r NI}{2\pi R_m}(R_e - R_i)$

a) $\bar{H} = \dfrac{NI}{2\pi r}$ $\qquad B = \dfrac{\mu_0\mu_r NI}{2\pi r}$ $\qquad \Phi_B(S) = \dfrac{h\mu_0\mu_r NI \ln(R_e/R_i)}{2\pi}$

2)



Seccion delgada $\rightsquigarrow$ l_m

Ley de Ampere generalizada $\qquad \oint_C \bar{H}\cdot d\bar{l} = I_c \rightsquigarrow H l_m = NI$

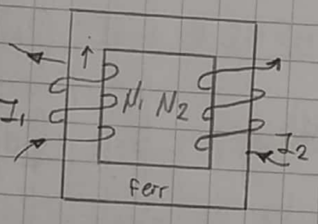
Seccion lineal: $\vec{B} = \mu_0\mu_r\vec{H}$

$$\boxed{B = \frac{\mu_0\mu_r NI}{l_m}} \qquad \boxed{H = NI/l_m}$$

b) $B = 0{,}1\ T \rightsquigarrow \boxed{I = 0{,}16\ A}$

$$\boxed{B = 0{,}1\ T} \qquad \boxed{H = 80\ A/m}$$

3)



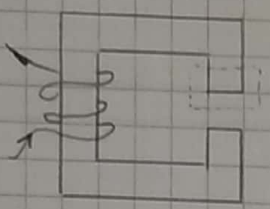
Ley de Ampere generalizada $\qquad \oint_C \vec{H}\cdot d\bar{l} = I_c$

$$\oint_C \bar{H}\cdot d\bar{l} = N_1 I_1 - N_2 I_2 = H\cdot l_m \rightsquigarrow H = \frac{N_1 I_1 - N_2 I_2}{l_m}$$

$$B = \frac{\mu_0\mu_r}{l_m}(N_1 I_1 - N_2 I_2) \qquad M = \frac{B}{\mu_0} - H = \frac{N_1 I_1 - N_2 I_2}{l_m}(\mu_r - 1)$$

$$\boxed{H = -2420\ A/m} \quad \boxed{B = -3{,}04\ T} \quad \boxed{M = 2417580\ A/m}$$

42)



Ley de Ampere generalizada $\curvearrowright \oint_c \bar{H} \cdot d\bar{l} = I_c$

$$H_m(l_m - l_e) + H_e\, l_e = NI$$

$$\Phi_B(S) = B_e \cdot S = B_m S \rightsquigarrow B_e = B_m = B$$

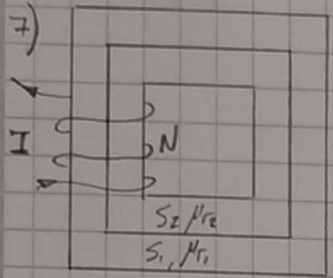
$$\frac{B}{\mu_0\mu_r}(l_m - l_e) + \frac{B}{\mu_0}\, l_e = NI \qquad \boxed{H_m = 79{,}6 \ A/m}$$

$$\Rightarrow \boxed{I = 0{,}56\,A} \qquad \boxed{H_e = 79600 \ A/m}$$

$$\mu_0 H_e = \mu_0 \mu_r H_m$$

$$H_e = \mu_r H_m$$

7)



$$\oint_c \bar{H} \cdot d\bar{l} = I_v|_{conc}$$

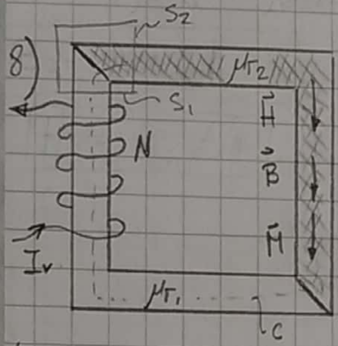
$$H_1 l_m = NI = H_2 l_m \quad (1) \qquad H_1 = H_2 = H = 700 \ A/m$$

$$B_1 = \mu_0 \mu_{r_1} H_1 \qquad B_2 = \mu_0 \mu_r H_2 \quad (2)$$

$$\Phi_B(S) = B_1 S_1 = B_2 S_2 \quad (3)$$

(2) $B_1 = 0{,}88\ T$ $\qquad B_2 = 1{,}76\ T$ $\qquad M = \dfrac{B}{\mu_0} - H \ \begin{cases} M_1 = 699581{,}75 \ A/m \\ M_2 = 1.399.863{,}5 \ A/m \end{cases}$

8)



$\left. \begin{array}{l} L_m = 50\ cm \\ S = 1\ cm^2 \end{array} \right\}$ $\sqrt{S} \ll L_m \Rightarrow$ Seccion delgada

* Materiales: • 1- Ferromagnetico $\mu_{r_1} = 1000$

$N = 350$ • 2- Ferromagnetico $\mu_{r_2} = 2000$

$I = 1A$

Ley de Ampere generalizada

$$\oint_c \vec{H} \cdot d\vec{l} = H_1 L_{m_1} + H_2 L_{m_2} = I_v|_{conc} = N I_v \rightsquigarrow H_1 L_{m_1} + H_2 L_{m_2} = N I_v \quad (1)$$

Circ horaria (+)

Ley de Gauss

$$\oiint_\Sigma \bar{B} \cdot d\bar{s} = -B_1 S_1 + B_2 S_2 = 0 \rightsquigarrow B_1 S_1 = B_2 S_2 \rightsquigarrow B_1 = B_2 \ (2), \quad S_1 = S_2$$

Secc delg $\rightsquigarrow \vec{B}$ unif $\wedge$ cte

Condicion de frontera

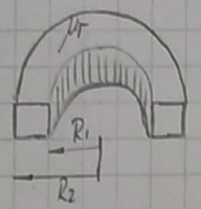
$$B_{1N} = B_{2N} \checkmark$$

Suponiendo los materiales lineales: $B = \mu_0 \mu_r H \ (3) \ \wedge \ M = \dfrac{B}{\mu_0} - H \ (4)$

$$\frac{B L_{m_1}}{\mu_0 \mu_{r_1}} + \frac{B L_{m_2}}{\mu_0 \mu_{r_2}} = N I_v \rightsquigarrow B = 1{,}17\ T \qquad H_2 = 466{,}67\ A/m \qquad M_2 = 932866{,}67\ A/m$$

$$H_1 = 933{,}3\ A/m \qquad M_1 = 932400\ A/m$$

9)



$R_1 = 0{,}11\ m$ $\qquad N = 2000$

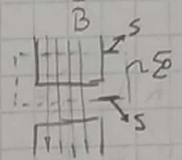
$R_2 = 0{,}12\ m$ $\qquad I_v = ?$

$\mu_r =$ No lineal $\rightsquigarrow$ Graf (B vs H)

a) Ley de Ampere generalizada $\oint_C \vec{H} \cdot \overline{dl} = I_v|_{conc} \rightsquigarrow H\, 2\pi R_M = N I_v \rightsquigarrow H = \dfrac{N I_v}{2\pi R_m}$

Segun grafico, si $B = 1\ T \Rightarrow H = 80\ A/m \rightsquigarrow \dfrac{H\, 2\pi R_M}{N} = I_v \rightsquigarrow \boxed{I_v = 0{,}029\ A}$

b) Se realiza un entrehierro de $e = 0{,}001\ m$



Se asume que el campo de induccion no se escape en el entrehierro

Ley de Ampere generalizada $\oint_C \vec{H} \cdot \overline{dl} = I_v|_{conc} \rightsquigarrow H_m(2\pi R_M - e) + H_e \cdot e = N I_v$ (1)

Ley de Gauss $\oiint_\Sigma \overline{B} \cdot d\overline{s} = 0$, Seccion delgada $\rightsquigarrow B_e \cdot S = B_m \cdot S$, $B_m = B_e$

$B_m = B_e = 1T \rightsquigarrow \begin{cases} H_m = 80\ A/m \\ H_e = \dfrac{B_e}{\mu_0} = 795.774{,}71\ A/m \end{cases}$ de (1) $I_v = 0{,}427 A$

c). Material Hipernick

a') $I_v = \dfrac{H\, 2\pi R_M}{N}$, $B = 1\ T \Rightarrow H = 72\ A/m \rightsquigarrow I_v = 0{,}026\ A$

b') $\left.\begin{array}{l} H_e = 795.\cancel{774}{,}74\ A/m \\ H_m = 72\ A/m \end{array}\right\} I_v = 0{,}424\ A$

. Material Permendur

$B = 1 \Rightarrow H_m = ?$

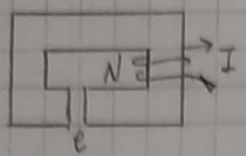
H	—	B
112	—	0,950
120	—	1,085
8	—	0,135
x	—	0,05

$\rightsquigarrow x = 2{,}96 \Rightarrow H = 114{,}96\ A/m$

a'') $I_v = 0{,}041\ A$

b'') $I_v = 0{,}44\ A$

12)



$L_m = 2\,m$; $S = 1\,cm^2$ } $\sqrt{S} << L_m \leadsto$ Seccion delgada

$N = 500$; $e = 0{,}001\,m$; $B_e = 1T$; $I_v = ?$

Ley de Ampere generalizada $\oint_C \bar{H} \cdot d\bar{l} = I_v|_{conc}$

$$H_e \cdot e + H_m (L_m - e) = N I_v \qquad (1)$$

Ley de Gauss $\oiint_S \bar{B} \cdot d\bar{S} = 0 \leadsto S_e = S_m \Rightarrow B_e = B_m = 1T$

$H_m \cong 200\ A/m$ x gráfico B vs H de Acero al Silicio

$H_e = \frac{B_e}{\mu_0} = 795.774{,}75\ A/m$

} a (1) $\leadsto I_v = 2{,}39\ A$

13) $\frac{B \cdot 0{,}001}{\mu_0} + H_m\, 1{,}999 = 750$

$$B = (750 - H_m \cdot 1{,}999) \cdot 1{,}256 \cdot 10^{-3}$$

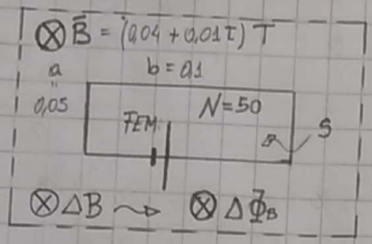
$$B = 0{,}942 - 0{,}00251\, H_m$$

$\begin{cases} H = 0 & B = 0{,}94\ T \\ B = 0 & H = 375{,}2\ A/m \end{cases}$

Punto de trabajo : $\begin{bmatrix} B = 0{,}65\ T \\ H_m = 90\ A/m \end{bmatrix}$

Scanned by CamScanner

-1)

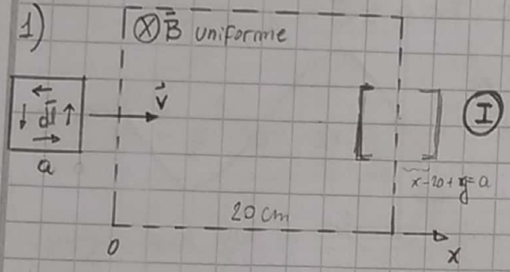


$$\iint_S \bar{B} \cdot d\bar{s} = B \cdot a \cdot b \cdot N = \Phi_B, \quad \vec{B} \parallel d\vec{s}$$

$$FEM = -\frac{d\Phi_B}{dt}\Big|_{\text{Cante}} = \frac{-d(0{,}04 + 0{,}01t)\cdot a \cdot b \cdot N}{dt} = -a \cdot b \cdot 0{,}01 \cdot N$$

$$FEM = -2{,}5 \cdot 10^{-3}\ V = -2{,}5\ mV$$

1)



$$\Phi_B = \iint_S \bar{B} \cdot d\bar{s}, \quad d\bar{s} \parallel \vec{B}$$

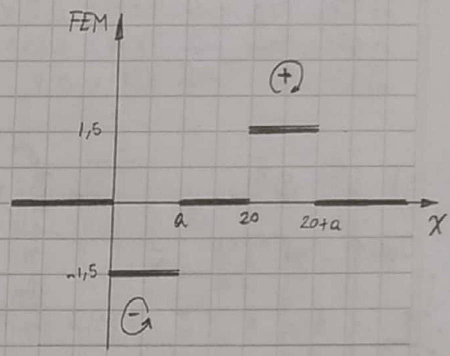
$x < 0 \rightsquigarrow \Phi_B = 0$

$0 < x < a \rightsquigarrow \Phi_B = \int_0^a \int_0^{vt} B \cdot ds = B \cdot a \cdot vt$

$a < x < 20 \rightsquigarrow \Phi_B = \int_0^a \int_0^a \bar{B} \cdot d\bar{s} = a^2 B$

$20 < x < 20 + a \rightsquigarrow \Phi_B = \int_0^a \int_{20+vt}^{20} \bar{B} \cdot d\bar{s} = -B\, a\, vt$

$x > 20 + a \rightsquigarrow \Phi_B = 0$



Siendo $N = 50$ esp

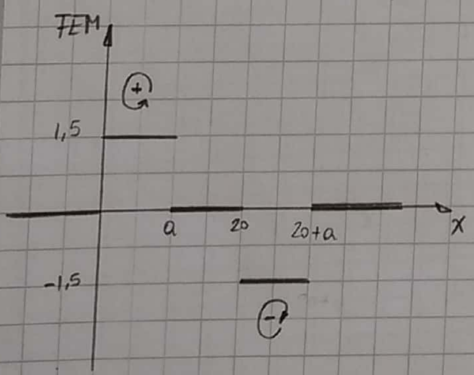
$$\Phi_B = \begin{cases} 0 \\ N a B v t \\ N a^2 B \\ -N a B v t \\ 0 \end{cases} \qquad FEM = \begin{cases} 0 & x < 0 \\ -N a B v & 0 < x < a \\ 0 & a < x < 20 \\ +N a B v & 20 < x < 20 + a \\ 0 & 20 + a < x \end{cases} = \begin{cases} 0 \\ -1{,}5\ V \\ 0 \\ +1{,}5\ V \\ 0 \end{cases}$$

II

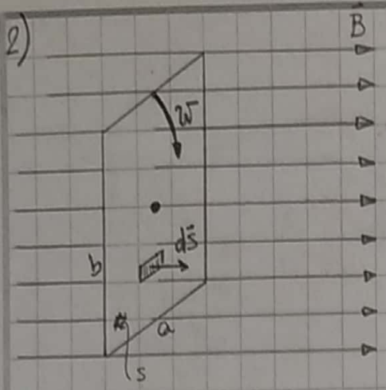
$$FEM = \oint_C \bar{V} \times \bar{B} \cdot d\bar{l} = \begin{cases} 0 & x < 0 \\ |V||B| \cdot a \cdot N = 1{,}5\ V & 0 < x < a \\ 0 & a < x < 20 \\ -|V||B| \cdot a\, N = -1{,}5\ V & 20 < x < 20 + a \\ 0 & 20 + a < x \end{cases}$$

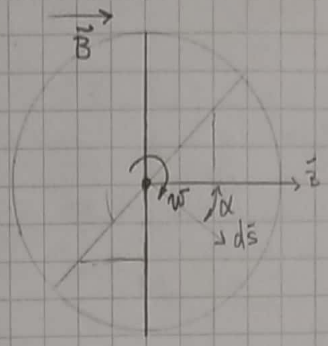
$\bar{V} \times \vec{B} = \uparrow$

$d\bar{l} = \downarrow \overset{\leftarrow}{\underset{\rightarrow}{}} \uparrow$



2)

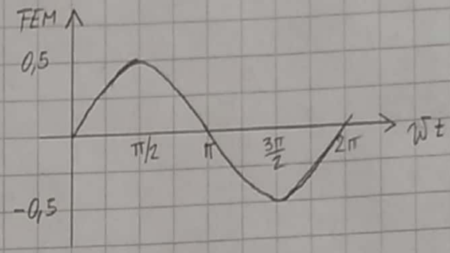




$W = 1500 \text{ rpm} \cdot \frac{2\pi}{\text{rpm}} \cdot \frac{1 \text{ min}}{60 \text{ seg}}$

$W = 50\pi \frac{1}{seg}$

$$\Phi_B = N \iint_S \bar{B} \cdot d\bar{s} = N \iint_S |\bar{B}| |d\bar{s}| \cos\alpha = N B\, a \cdot b \cos\alpha = N B\, a b \cos Wt \quad , \alpha = Wt$$



Ley de Faraday - Lentz

$$FEM = -\frac{d\Phi_B}{dt} = N B\, a b \sin Wt = 0{,}5 \sin(50\pi t)$$

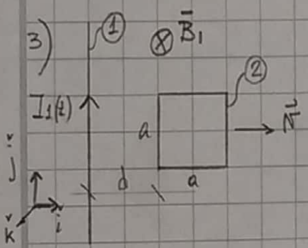
i) $Wt = \pi/2 \rightsquigarrow FEM = 0{,}5\ V$

ii) $Wt = 3\pi/4 \rightsquigarrow FEM = 0{,}353\ V$

iii) $Wt = \pi \rightsquigarrow FEM = 0\ V$

$B\, 2\pi r = I_1 \mu_0$

3)



$$\vec{B}_1 = \frac{\mu_0 I_1(t)}{2\pi d} (-\check{k})$$

$$\Phi_{B_{12}} = N \iint_{S_2} \vec{B}_1 \cdot d\bar{s}_2 = N a \int_d^{d+a} \frac{\mu_0 I}{2\pi x} dx, \quad d\bar{s} \parallel \vec{B}$$

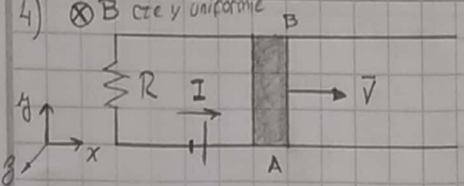
$$\Phi_{B_{12}} = \frac{N \mu_0 I_1}{2\pi} \cdot \ln\left(\frac{d+a}{d}\right) \quad a) \quad M = \frac{\Phi_{B_{12}}}{I_1} = \frac{N\mu_0}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

b) $x_0 = d$ $$\Phi_{B_{12}} = \int_0^a \int_{Vt}^{Vt+a} \frac{N \mu_0 I_1}{2\pi x} dx\, dy = \frac{N \mu_0 I_1 a}{2\pi} \ln\left(\frac{a+Vt}{Vt}\right)$$

Ley de Faraday Lentz

$$FEM = -\frac{d\Phi_{B_{12}}}{dt} = \frac{N\mu_0 a}{2\pi} \left[-\frac{dI(t)}{dt} \left(\ln \frac{a+Vt}{Vt} \right) - \left(\frac{V}{a+Vt} - \frac{1}{t} \right) \cdot I \right)$$

4) $\otimes \vec{B}$ cte y uniforme



a) $\Phi_B = \int_A^B \int_0^{vt} B \, dx \, dy = B v t (B-A) = B v t L \qquad d\vec{s} = ds(-\check{k})$

$FEM = -\frac{d\Phi_B}{dt} = -BvL = -2V \rightsquigarrow$ Ley de Ohm: $RI = \Delta V \rightsquigarrow I = 0{,}2 A$

b) $\vec{F} = \int_A^B I \, d\vec{l} \times \vec{B} = \int_0^L IB \, dl \, (-\check{x}) = -IBL\check{x} = -0{,}04 \, N \, \check{x}$

La fuerza para que la velocidad se mantenga constante debería ser $\vec{F} = +0{,}04 \, N \, \check{x}$

c) $Pot_{dis} = \Delta V_R \cdot I = R \cdot I \cdot I = 0{,}4 \, W$

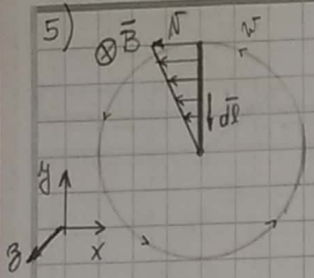
$Pot_{ent} = F \cdot v = 0{,}4 \, W \qquad W = N \cdot \frac{m}{seg} = \frac{J}{seg}$

d) $F = IBL \quad , \quad I = -\frac{BvL}{R} \rightsquigarrow F = -\frac{B^2 L^2 v}{R} = m \frac{dV}{dt}$

$-\frac{B^2L^2}{Rm} dt = \frac{1}{v} dv \rightsquigarrow -\frac{B^2L^2}{Rm} \cdot t = \ln v \rightsquigarrow v(t) = e^{-\frac{B^2L^2}{Rm} \cdot t}$

Se va frenando. Si $t \to \infty$, $v(t \to \infty) \cong 0$

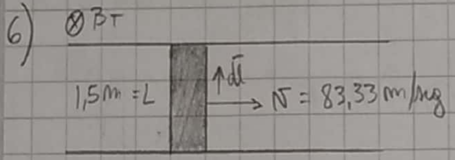
5)



$v = -w \cdot y \qquad \bar{dl} = -dy\,\hat{j}$

$$FEM = \oint_c \bar{v} \times \bar{B} \cdot \bar{dl} = \int_c vB\,dl = \int_c w\,y\,B\,dy = \frac{wBL^2}{2} = 0{,}17\,V$$

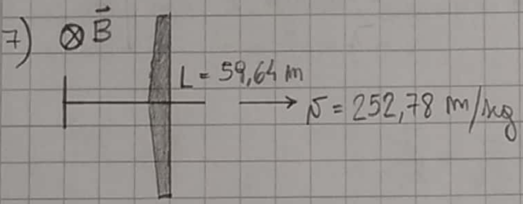
6)



$$\int_c \bar{V} \times \bar{B} \cdot \bar{dl} = VB_TL = 0{,}00625 = FEM$$

$B_{terrestre} = 0{,}00005\ T$

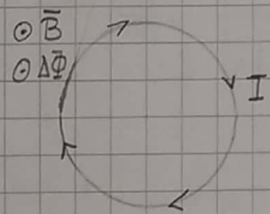
7)



$$\vec{B} = \mu_0 \vec{H} = 6{,}28 \cdot 10^{-6}\ T$$

$$\int_c \bar{V} \times \bar{B} \cdot \bar{dl} = VBL = 0{,}0947\ V = FEM$$

8) a) Sí, aumenta el flujo por lo tanto se induce una corriente:



b) $B = 0{,}3\ T$

$D_0 = 0{,}1\ m \qquad r_0 = 0{,}05$

$D_f = 0{,}2\ m \qquad r_f = 0{,}1$

$t = 0{,}04\ sg$

$v = 2{,}5\ m/sg$

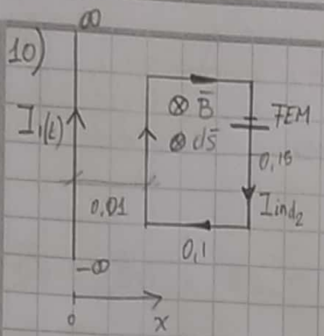
$$\iint_S \bar{B} \cdot \bar{ds} = B \int_0^{2\pi} \int_{0{,}05}^{vt} r\,dr\,d\varphi = \left[(vt)^2 - 0{,}05^2\right]\pi = \Phi_B$$

$$FEM = -\frac{d\Phi_B}{dt} = -2v^2t\pi$$

$t_0 = 0 \rightsquigarrow FEM = 0$

$t_f = 0{,}04\ sg \rightsquigarrow FEM = -1{,}57\ V$

10)



$I_1(t) = 5 \cos(9t)$

$\bar{B}_1 = \dfrac{\mu_0 I_1(t)}{2\pi x}$

$$\Phi_{12} = \iint_S \frac{\mu_0 I(t)}{2\pi x}\, dx\, dy = \frac{\mu_0 I(t)}{2\pi}\, 0{,}15 \cdot \ln\left(\frac{0{,}1+d}{d}\right)$$

$$\Phi_{12} = \frac{\mu_0\, 5 \cos 9t \cdot 0{,}15}{2\pi} \ln\left(\frac{0{,}1+d}{d}\right) \qquad (1)$$

$$FEM = \frac{\mu_0\, 5 \sin 9t \cdot 0{,}15}{2\pi} \ln\left(\frac{0{,}1+d}{d}\right) \qquad (2)$$

a) $M = \dfrac{\Phi_{12}}{I_1} = \dfrac{\mu_0\, 0{,}15}{2\pi} \ln\left(\dfrac{0{,}1+d}{d}\right) = 7{,}2 \cdot 10^{-8}$ Henry

b) de (2) $FEM = 3{,}6 \cdot 10^{-7} \cdot \sin(9t)$ V

Ley de Ohm $FEM = \Delta V = R I$, $R = 10\ \Omega$

$I_{ind_2} = 3{,}6 \cdot 10^{-8} \sin(9t)$ A

c) Todos los resultados se multiplicarían por el nº de espiras de la bobina

11) Calcular autoinductancia del toroide del ej. 1 con datos de del ej 5 (guía 6)

Considerando sección gruesa $\Phi_B = \dfrac{h \mu_0 \mu_r N I \ln(Re/Ri)}{2\pi}$

Autoinductancia $L = N \dfrac{\Phi_B}{I} = \dfrac{h \mu_0 \mu_r N^2 \ln(Re/Ri)}{2\pi}$, $\begin{cases} \mu_r = 800 \\ N = 300 \\ I = 1\,A \end{cases}$

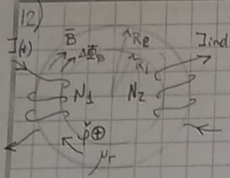
a) $R_i = 0{,}02$ m
$Re = 0{,}03$ m $\rightsquigarrow L = 5{,}84 \cdot 10^{-2}$ H
$h = 0{,}01$ m

b) $R_i = 0{,}029$ m
$Re = 0{,}03$ m $\rightsquigarrow L = 4{,}88 \cdot 10^{-2}$ H
$h = 0{,}1$ m

c) a' ~ $L = 1{,}44 \cdot 10^{-3}$ H En lugar de $\ln(Re/Ri)$ va $(Re - Ri)$

b' ~ $L = 1{,}44 \cdot 10^{-3}$ H

12)



$\int_C \bar{H}\cdot d\bar{l} = N_1 I(t)\ \mu_0 \rightsquigarrow H = \dfrac{N_1 I(t)\ \mu_0}{2\pi R_M}$ (Seccion delgada)

$B = \dfrac{\mu_0 \mu_r N_1 I(t)}{2\pi R_M}$ Mat lineal

$\Phi_B(S)_1 = \dfrac{\mu_0 \mu_r N_1 I_1(t)\ S}{2\pi R_M}$

$\Phi_{11} = N_1\, \Phi_B(S)_1 \rightsquigarrow L_1 = \dfrac{\Phi_{11}}{I_1} = \dfrac{\mu_0 \mu_r N_1^2 S}{2\pi R_M} = 0{,}08\ H$

$M = \dfrac{\mu_0 \mu_r N_1 N_2 S}{2\pi R_M} = 0{,}032\ H$

$\Phi_{22} = N_2\, \Phi_B(S)_2 \rightsquigarrow L_2 = \dfrac{\mu_0 \mu_r N_2^2 S}{2\pi R_M} = 0{,}013\ H$

Suponiendo que por ② circula una corriente I_2 (*)

(*) Sino $M^2 = L_1 L_2 \rightsquigarrow L_2 = \dfrac{M^2}{L_1} = \dfrac{0{,}032^2}{0{,}08} = 0{,}013\ H$ ✓

$FEM = -\dfrac{\mu_0 \mu_r N_1 N_2 . S}{2\pi R_M} . \underbrace{0{,}2}_{\frac{\partial I}{\partial t}} = 6{,}4 . 10^{-3}\ V = \dfrac{-d\Phi_{12}}{dt}$

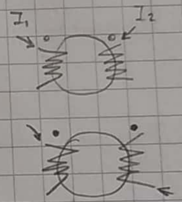
13) $I_1 = 20\ A \rightsquigarrow \vec{B}$: ↺

1) $I_2 = 2\ A \rightsquigarrow \vec{B}$: ↺

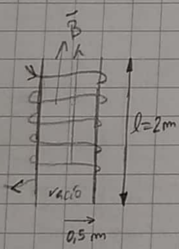
2) $I_2 = 2\ A \rightsquigarrow \vec{B}$: ↻

1) $U = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} + M I_1 I_2 = 17{,}3\ J$

2) $U = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} - M I_1 I_2 = 14{,}75\ J$



14)



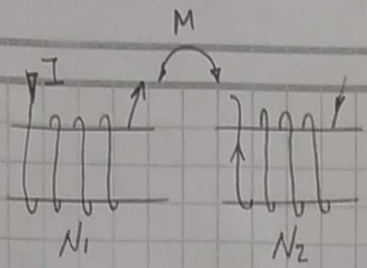
$B\, \cancel{a} = \mu_0 \dfrac{N}{L}\, \cancel{a} . I$ $\quad B = \mu_0 \dfrac{N I}{L} \rightsquigarrow I = 238{,}7\ A$

$\Phi_B = B . S = 1{,}5\, T\, \pi\, (1m)^2 = 4{,}71\ Wb$

$L = \dfrac{\Phi_B}{I} = 0{,}0197$

$U = \dfrac{L I^2}{2} = 562{,}5\ J$

$R = 0{,}2\ \Omega \rightsquigarrow \Delta V = R I = 47{,}75\ V \rightsquigarrow Pot_{dis} = \Delta V\, I = 11398{,}6\ Watt$

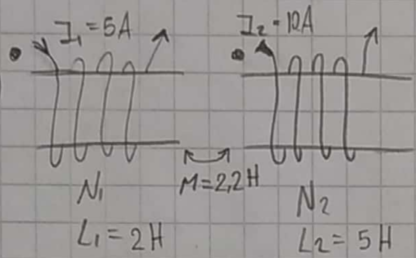
15) 

Acoplamiento perfecto

$\Phi_2 = \Phi_{21} + \Phi_{22} = -M I_1 + 0 = -M I_1 = -1{,}5 \cdot (2 + 0{,}5t)$

$e_2 = -M \dfrac{dI_1}{dt} = -1{,}5 \cdot 0{,}5 \ V = -0{,}75 \ V$

$U = \dfrac{L I_1^2}{2}$ ⇝ Energía total almacenada

16) 

$U = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} + M I_1 I_2 = 385 \ J$

Dado que las corrientes ingresan por bordes homologos (+)

Si las corrientes ingresaran por bordes no homologos (I_2 por el otro extremo) ⇝ Agregado

$U = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} - M I_1 I_2 = 165 \ J$

b) Los dos solenoides estan alejados entre ellos.

$U = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} = 275 \ J$

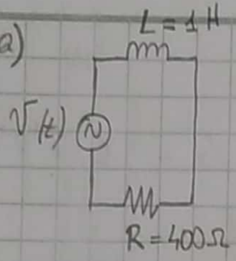
c) El trabajo es la dif. de energía entre las posiciones inicial y final.

$W^{\infty \to x} = U_F - U_0 = \dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} + M I_1 I_2 - \left[\dfrac{L_1 I_1^2}{2} + \dfrac{L_2 I_2^2}{2} \right] = M I_1 I_2 = 110 \ J$

Si la I_2 ingresara por el otro extremo:

$W^{\infty \to x} = -110 \ J$

1) a)



$V(t) = 311 \cos(wt)$

$f = 50\ Hz$

$w = 2\pi f$

$C \to \infty$

$\mathbb{V} = \mathbb{I}\, Z_{eq}$, $Z_{eq} = Z_R + Z_L = R + j X_L = R + j\, wL$, Se trabajará con valores pico.

$\mathbb{I} = \dfrac{\mathbb{V}}{\mathbb{Z}}$ $\quad \mathbb{V} = 311\, e^{j\cdot 0} \;\Rightarrow\; \varphi_V = 0 = \varphi_Z + \varphi_I$

$\mathbb{Z} = \sqrt{R^2 + (wL)^2}\; e^{j\varphi_Z}$, $\varphi_Z = \operatorname{atg}\left(\dfrac{wL}{R}\right) = 0{,}666$

$\mathbb{Z} = 508{,}6\, e^{j0{,}666}$

$\mathbb{I} = 0{,}611\, e^{-j0{,}666}$

Luego a) $\boxed{i(t) = 0{,}611 \cos(wt - 0{,}666)}$

$\mathbb{V}_R = \mathbb{I} \cdot Z_R = \mathbb{I}\, R = 244{,}4\, e^{-j\,0{,}666}\ \mathbb{V}$

$\mathbb{V}_L = \mathbb{I} \cdot Z_L = 0{,}611\, e^{-j\cdot 0{,}666} \cdot e^{j1{,}57} \cdot 314{,}2 = 191{,}9 \cdot e^{j0{,}9}$ $\qquad 1{,}57 = \pi/2 \;\alpha\; 90°$

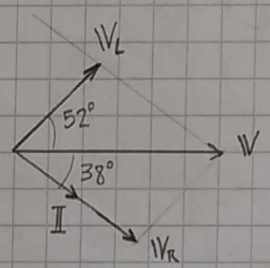
b) $V_R(t) = 244{,}4 \cos(wt - 0{,}666)$

$V_L(t) = 191{,}9 \cos(wt + 0{,}9)$

Potencia instantanea:

c) $P(t) = V(t) \cdot i(t) = 190 \cos(wt) \cdot \cos(wt - 0{,}666)$

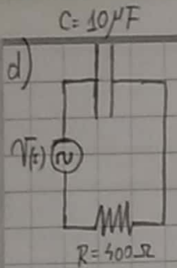
Diagrama fasorial



$\dfrac{0{,}9 \cdot 180}{\pi} = 52°$

$\dfrac{0{,}666 \cdot 180}{\pi} = 38°$

d)



$V(t) = 311 \cos(wt)$

$\mathbb{V} = Z_{eq} \cdot \mathbb{I}$

$\varphi_V = \varphi_Z + \varphi_I$, supongo $\varphi_V = 0 \Rightarrow \varphi_Z = -\varphi_I$

$$\mathbb{V} = \mathbb{V}_R + \mathbb{V}_C = Z_R \mathbb{I} + Z_C \mathbb{I} = X_R \mathbb{I} - jX_C \mathbb{I} = \mathbb{I}(X_R - jX_C) = \mathbb{I} Z_{eq}$$

- $Z_{eq} = X_R - jX_C = R - j\frac{1}{wC} = Z e^{j\varphi_Z} = 511,2\, e^{j \cdot 38,51^\circ}$

$Z = (X_R^2 + X_C^2)^{1/2} = \left(400^2 + \left[1/(2\pi \cdot 50 \cdot 10 \cdot 10^{-6})\right]^2\right)^{1/2} = 511,2\ \Omega$

$\varphi_Z = \arctan \frac{X_C}{X_R} = 38,51^\circ$

- $\mathbb{I} = \frac{\mathbb{V}}{Z_{eq}} = \frac{311\, e^{j \cdot 0}}{511,2\, e^{j 38,51^\circ}} = 0,608\, e^{j(-38,51^\circ)} \rightsquigarrow i(t) = 0,608 \cos(wt - 38,51^\circ)$

- $\mathbb{V}_R = Z_R \mathbb{I} = X_R \mathbb{I} = R\mathbb{I} = 243,2\, e^{j(-38,51^\circ)}$

$V_R(t) = 243,2 \cos(wt - 38,51^\circ)$

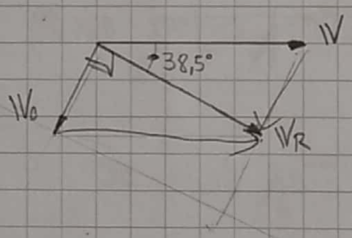
- $\mathbb{V}_C = Z_C \mathbb{I} = -jX_C \mathbb{I} = e^{-j(90)} \frac{1}{wC}\, 0,608\, e^{j(-38,51^\circ)} = 193,5\, e^{j(-128,51^\circ)}$

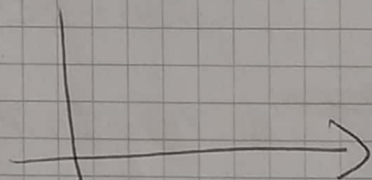
$V_C(t) = 193,5 \cos(wt - 128,51^\circ)$

Potencia instantanea:

$P(t) = V(t) \cdot i(t) = 189,1 \cos(wt - 38,51^\circ) \cdot \cos(wt)$

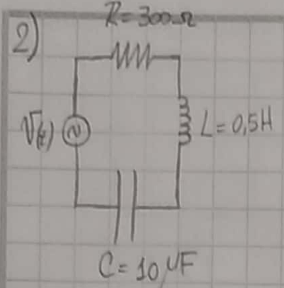
Diagrama fasorial:





$V_R^2 + V_C^2 = V^2$

2)



$i(t) = 5{,}2 \cos(\omega t + 60°)$, $\omega = 100$

$\rightarrow \mathbb{I} = 5{,}2\, e^{j60°}$

$\varphi_V = \varphi_I + \varphi_Z = 0 \rightarrow$

$V(t) = R\, i(t) + L \frac{di(t)}{dt} + \frac{\int i(t)\, dt}{C} = 5{,}2 \cos(\omega t + 60°)\, 300 = 1560 \cos\alpha + 4940 \sin\alpha$

$- 5{,}2 \sin(\omega t + 60)\, 0{,}5\, \omega$

$+ \frac{5{,}2 \sin(\omega t + 60)}{\omega C}$

$Z_{eq} = Z\, e^{j\varphi_Z}$

$\boxed{\mathbb{V} = \mathbb{I}\, Z_{eq}} = 5{,}2\, e^{j60°} \cdot (300 + j(50 - 1000)) = 5180\, e^{j(-12{,}5°)}$

$Z = 996{,}24$

$\varphi_Z = -72{,}5°$

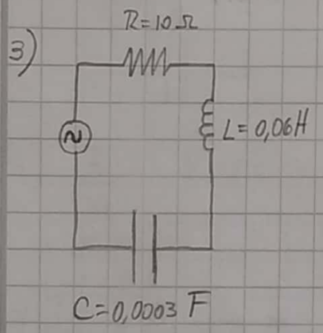
Potencia instantanea : $P(t) = V(t) \cdot i(t) = 5{,}2 \cos\alpha \cdot (1560 \cos\alpha + 4940 \sin\alpha)$

Hallar C / el circuito quede en resonancia.

$L \frac{di(t)}{dt} = \frac{\int i(t)\, dt}{C} \rightarrow \cancel{5{,}2 \sin(\omega t + 60°)}\, L\, \omega = \frac{\cancel{5{,}2 \sin(\omega t + 60°)}}{\omega C}$

$C = \frac{1}{\omega^2 L} = 200\, \mu F$

3)



$f = 50\, Hz$

Calculo de reactancias :
$$\begin{cases} X_R = R = 10\, \Omega \\ X_L = \omega L = 18{,}85\, \Omega \\ X_C = (\omega C)^{-1} = 10{,}61\, \Omega \end{cases}$$

$V(t) = 311 \cos(\omega t)$

Calculo de impedancias :
$$\begin{cases} Z_R = X_R = 10\, \Omega \\ Z_L = j X_L = j\, 18{,}85\, \Omega \\ Z_C = -j X_C = -j \cdot 10{,}61\, \Omega \end{cases}$$

$Z_{eq} = 10 + 8{,}24\, j = 12{,}96\, e^{j\, 39{,}5°}$

$Z = 12{,}96$

$\varphi_Z = 39{,}5°$

$X_L > X_C \rightarrow$ Circuito con comportamiento inductivo

b) $\mathbb{V} = 311\, e^{j \cdot 0} = \mathbb{I} \cdot Z_{eq} \rightarrow \mathbb{I} = \frac{311\, e^{j0°}}{12{,}96\, e^{j39{,}5°}} = 24\, e^{j(-39{,}5°)}$ A

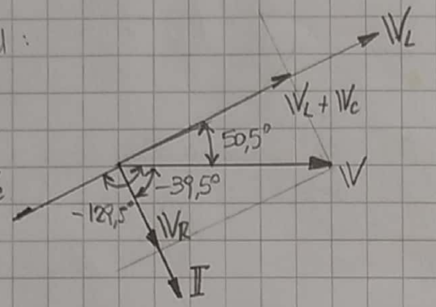
$\mathbb{I} = 24 \cos(39{,}5°) + j\, 24 \sin(39{,}5°) = (18{,}5 - 15{,}3\, j)$ A

$\mathbb{V}_R = \mathbb{I} . Z_R = 24 . e^{j(-39,5°)} . 10 = 240 . e^{j(-39,5°)}$ V

$\mathbb{V}_L = \mathbb{I} Z_L = 24 e^{j(-39,5°)} . 18,85 e^{j(90°)} = 452,4 e^{j(50,5°)}$ V

$\mathbb{V}_C = \mathbb{I} Z_C = 24 . e^{j(-39,5°)} . 10,61 e^{j(-90)} = 254,6 e^{j(-129,5°)}$ V

Diagrama fasorial:



Calculo de potencias:

- Potencia activa = $I_{ef} . V_{ef} \cos(\varphi_Z) = I_{ef}^2 . R = 2880$ W ✓
- Potencia reactiva = $I_{ef} V_{ef} \sin(\varphi_Z) = I_{ef}^2 . X_{LC} = 2373$ W ✓
- Potencia aparente = $I_{ef} . V_{ef} = (P_{act}^2 + P_{react}^2)^{1/2} = 3732$ W ✓

Frecuencia de resonancia

$X_C = X_L$

$\frac{1}{WC} = WL \rightsquigarrow W = 235,7$

4)

$V_0 = 311\ V$ $\quad V_{ef} = 220\ V$ $\quad f = 50\ Hz$

$V_{0_C} = 169{,}7$ $\quad V_{ef_C} = 120$

a) $V_{ef}^2 = V_{ef_C}^2 + V_{ef_R}^2 \rightsquigarrow V_{ef_R} = 184{,}4\ V = I_{ef} \cdot R \rightsquigarrow \boxed{I_{ef} = 0{,}369\ A}$

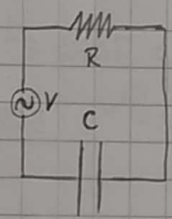
b) $V_{ef_C} = \frac{I_{ef}}{wC} \rightsquigarrow C = 9{,}79\ \mu F$

c) $P = I_{ef}^2 \cdot R = 68{,}1\ W$

d) $Q = \frac{I_{ef}^2}{wC} = 44{,}3\ W$

e) $S = (P^2 + Q^2)^{1/2} = 81{,}24\ W$

5)



$V_{ef} = 200\ V$

$f = 50\ Hz$

a) $V_{ef}^2 = V_{ef_C}^2 + V_{ef_R}^2$, $\quad V_{ef_R} = 150\ V$

$\rightarrow V_{ef_C} = 132{,}3\ V$

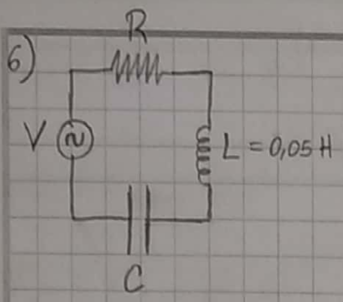
b) $\frac{R}{X_c} = R\,wC =$

c) $I_{ef} = 1\ A$

- $V_{ef_R} = I_{ef}\,R \rightsquigarrow R = 150\ \Omega$
- $V_{ef_C} = \frac{I_{ef}}{wC} \rightsquigarrow C = 24\ \mu F$

d) $\cos(\varphi_Z) = \cos\left[atg\left(\frac{X_c}{R}\right)\right] = \cos\left[atg\left(\frac{1}{wCR}\right)\right] = \begin{cases} R = 150\ \Omega \rightsquigarrow \cos\varphi = 0{,}75 \\ R = 300\ \Omega \rightsquigarrow \cos\varphi = 0{,}915 \end{cases}$

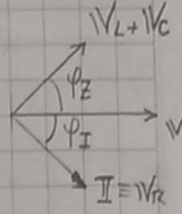
6)



$V_0 = 141,42\ V$ $\quad i(t) = 35,35 \cos(\omega t - 45°)$

$f = 50\ Hz$ $\quad \mathbb{I} = 35,35\ e^{j(-45°)}$

Sea $\varphi_V = 0 \rightsquigarrow \varphi_I = -\varphi_Z \rightsquigarrow \varphi_Z = 45°$



a) • $tg\ 45° = \dfrac{\omega L - 1/\omega C}{R}$ $\quad (1)$

• $V_0^2 = V_{0_R}^2 + V_{0_{LC}}^2 = I_0^2\left(R^2 + \left(L\omega - \dfrac{1}{\omega C}\right)^2\right)$ $\quad (2)$

de (1) $\quad R = \dfrac{\omega L - 1/\omega C}{tg\ 45°}$

a (2) $\quad \dfrac{V_0^2}{I_0^2} = 2\left(L\omega - \dfrac{1}{\omega C}\right)^2 \longrightarrow \dfrac{V_0}{I_0} = \sqrt{2}\left(L\omega - \dfrac{1}{\omega C}\right) \longrightarrow C = 247\ \mu F$

$\hookrightarrow 1 + (1/tg\ 45)^2 = 2$

Luego, de (1) $\quad R = 2,83\ \Omega$

b) $V_R = I_0 R = 70,75\ V \rightsquigarrow \mathbb{V}_R = \mathbb{I} R = 35,35 . e^{j(-45)} . 2,83. = 70,75\ e^{j(-45)}$

$V_L = I_0 \omega L = 555,28\ V \rightsquigarrow \mathbb{V}_L = \mathbb{I} . Z_L = 35,35\ e^{j(-45)} . 15,71\ e^{j(90)} = 555,28\ e^{j45}$

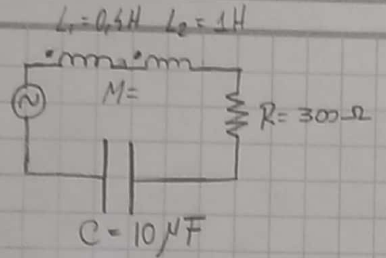
$V_C = \dfrac{I_0}{\omega C} = 454,28\ V \rightsquigarrow \mathbb{V}_C = 454,28\ e^{j(-135)}$

c) $P = I_{ef}^2 . R = I_{ef} V_{ef} \cos \varphi_Z = 1768\ W$; $Z = 2,83 + j\ 2,83$ $\quad \varphi_Z = 45°$

$Q = I_{ef}^2 X_{LC} = I_{ef} V_{ef} \sin \varphi_Z = 1768\ W$

$S = I_{ef} V_{ef} = \sqrt{P^2 + Q^2} = 2500\ W$

7)

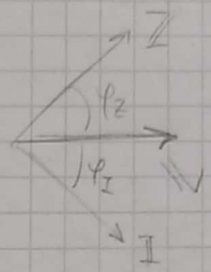


$V_{ef} = 220V$

$V_0 = 311V$

$f = 50 Hz$

$M = K\sqrt{L_1 L_2} = \frac{\sqrt{L_1 L_2}}{1+d^2}$



$\varphi_I = -45° \Rightarrow \varphi_V = 0 \Rightarrow \varphi_Z = 45°$

$L_T = L_1 + L_2 \pm 2M \begin{cases} L_{T_1}^{\ominus} = L_1 + L_2 - M \\ L_{T_2}^{\oplus} = L_1 + L_2 + M \end{cases}$

$Z = R + j\left(wL_T - \frac{1}{wC}\right) \begin{cases} Z_1 \\ Z_2 \end{cases}$

$tg\, 45 = \left(\frac{wL_{T_1} - 1/wC}{R}\right) \rightarrow d_1 = -3,22 \text{ cm}$

$\rightarrow d_2 = 1,23 \text{ cm}$

Se usa (2) que es $\oplus$ $\Rightarrow$ Bornes homólogos

Calorimetría

1) Fe: $m = 1\ Kg$ $Q = Ce\ m . \Delta T$

$Ce = 481\ \frac{J}{Kg\ °C}$ $\boxed{Q = 38480\ J}$

$\Delta T = 80°C$

H_2O: $Ce = 4180\ \frac{J}{Kg\ °C}$ $\boxed{Q = 334400\ J}$

2) Capacidad calorífica: $C = Ce . m$

$Ce_{Al}\ m_{Al} = Ce_{H_2O}\ m_{H_2O}$, $m_{H_2O} = 1\ Kg$; $Ce_{H_2O} = 4180\ \frac{J}{Kg\ °C}$; $Ce_{Al} = 920\ \frac{J}{Kg\ °C}$

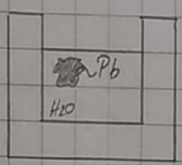
$\boxed{m_{Al} = 4,54\ Kg}$

3) H_2O; Situacion inicial: solido a $-20°C$ $m = 2\ Kg$

Situacion final: vapor a $100°C$

$Q = Ce_{Hielo}\ m . \Delta T + Cl_{Hielo}\ m + Ce_{H_2O}\ m\ \Delta T + Cl_{H_2O}\ m = 6\,098\,720\ J$

4) Sistema aislado

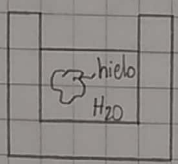


$m_{H_2O} = 0,5\ Kg$ $T_0 = 20°C$ $T_F = ?$

$m_{Pb} = 0,5\ Kg$ $T_0 = 100°C$

$Q = 0 \rightsquigarrow Ce_{H_2O}\ m_{H_2O} (T_F - 20°) + Ce_{Pb}\ m_{Pb} . (T_F - 100°) = 0 \rightsquigarrow \boxed{T_F = 22,42°C}$

6)



$m_{H_2O} = 1\ Kg$ $T_0 = 20°C$

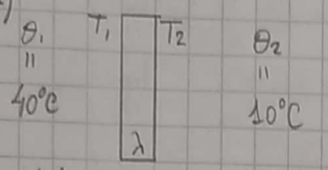
$m_{Hielo} = 1\ Kg$ $T_0 = -5°C$

$Q = 0 \rightsquigarrow Ce_{H_2O}\ m_{H_2O} (T_F - 20°C) + Cl_{Hielo}\ \Delta m_{hielo} + Ce_{hielo}\ m_{hielo} (0 - (-5°C)) = 0$

Si se hubiera derretido todo el hielo $\Rightarrow T_F = -62,13°C \rightsquigarrow$ Absurdo.

Luego, la $T_F = 0$ y $\Delta m_{hielo} = 0,22\ Kg \Rightarrow$ La masa restante de hielo es $0,78\ Kg$

1)



$h = 6{,}4\ \frac{W}{m^2\,°C}$ $\quad e = 1\,cm$ $\quad \lambda = 0{,}35\ \frac{W}{m\,°C}$

$$\dot{Q}_2 = -\lambda S \nabla T = -\lambda S \frac{dT}{dx} = \frac{\lambda S}{e}(T_1 - T_2)$$

$$\frac{\dot{Q}e}{\lambda S} = T_1 - T_2 \qquad (1)$$

$$\dot{Q}_1 = h_1 S(\theta_1 - T_1) \rightsquigarrow (\theta_1 - T_1) = \dot{Q}/h_1 S \qquad (2)$$

$$\dot{Q}_3 = h_2 S(T_2 - \theta_2) \rightsquigarrow (T_2 - \theta_2) = \dot{Q}/h_2 S \qquad (3)$$

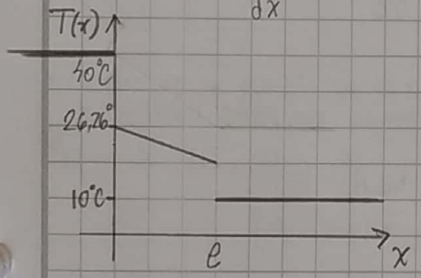
Luego (1) + (2) + (3) = $\theta_1 - \theta_2 = \dot{Q}\left(e/\lambda S + 1/h_1 S + 1/h_2 S\right)$

a) Por lo tanto $\frac{\dot{Q}}{S} = 87{,}96\ \frac{W}{m^2}$

b) de (2) $\quad T_1 = 26{,}26\,°C$

de (3) $\quad T_2 = 23{,}74\,°C$

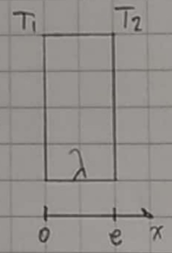
c) $\dot{Q}_2 = -\lambda S \frac{dT}{dx} \rightsquigarrow \dot{Q}x = \lambda S(-T(x) + T_1) \rightsquigarrow T(x) = -\frac{\dot{Q}x}{\lambda S} + T_1$



d) $\nabla T(x) = \frac{dT(x)}{dx} = -\frac{Q}{\lambda S}$

2) $S_V = 3\ m^2$	Ventana	$e_V = 0{,}004\ m$	Vidrio	$\lambda_V = 0{,}07\ \frac{W}{m\ °C}$	$h_V = h$	
$S_P = 2{,}5\ m^2$	Puerta	$e_P = 0{,}05\ m$	Madera	$\lambda_P = 0{,}17\ \frac{W}{m\ °C}$	$h_P = h$	$h = 6{,}4\ \frac{W}{m^2\ °C}$
$S_W = 29{,}5\ m^2$	Pared (Wall)	$e_W = 0{,}3\ m$	Ladrillo	$\lambda_W = 0{,}41\ \frac{W}{m\ °C}$	$h_W = h$	
$S_T = 12\ m^2$	Techo	$e_T = 0{,}1\ m$	Hormigon	$\lambda_T = 0{,}76\ \frac{W}{m\ °C}$	$h_T = h$	

$\theta_1 = 20°C$



$\theta_2 = 0°C$

$$\delta\dot{Q} = -\lambda \bar{\nabla} T \cdot d\bar{S} \rightsquigarrow \dot{Q} = -\lambda \frac{dT}{dx} S$$

$$\begin{cases} \bar{\nabla} T = \frac{dT}{dx} \check{x} \\ d\bar{S} = dS\ \check{x} \end{cases}$$

$$\dot{Q} = \frac{\lambda S}{e}(T_1 - T_2) \quad (1)$$

$$\dot{Q} = h S (\theta_1 - T_1) \quad (2)$$

$$\dot{Q} = h S (T_2 - \theta_2) \quad (3)$$

$$(\theta_1 - \theta_2) = \frac{\dot{Q}}{S}\left(\frac{e}{\lambda} + \frac{1}{h} + \frac{1}{h}\right)$$

$$\dot{Q} = \frac{(\theta_1 - \theta_2) \cdot S}{\left(\frac{e}{\lambda} + \frac{1}{h} + \frac{1}{h}\right)}$$

$\dot{Q}_V = 162{,}31\ W$

$\dot{Q}_P = 82{,}42\ W$

$\dot{Q}_W = 565{,}02\ W$

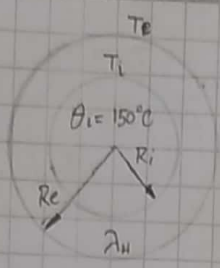
$\dot{Q}_T = 540{,}44\ W$

$\dot{Q}_{TOTAL} = 1350{,}19\ W$

La estufa debera entregar una potencia de 1350,19 W para mantener la habitacion a 20°C cuando afuera esta a 0°C

Al hacer la suposicion de plano infinito, se define que las superficies isotermas son planos infinitos y la distribucion de Calor es uniforme. Por ello se desprecian los efectos de borde

3)



$\theta_e = 20°C$

$R_i = 0{,}01\ m$ $\quad h_i = 581{,}5\ \frac{W}{m^2\,°C}$

$R_e = 0{,}03\ m$ $\quad h_e = 5{,}8\ \frac{W}{m^2\,°C}$

$e = 0{,}02\ m$ $\quad \lambda_H = 69{,}78\ \frac{W}{m\,°C}$

Se realiza la suposicion de superficie cilíndrica infinita, lo que implica que el calor esta distribuido uniformemente, esto es, las superficies isotermas son cilindros.

Conduccion: $\delta\dot{Q} = -\lambda\,\bar{\nabla}T \cdot d\bar{S}$, $\bar{\nabla}T \cdot d\bar{S} = \frac{dT}{dr}\check{r}\; ds\,\check{r} = \frac{dT}{dr}ds$

$\dot{Q}_1 = -\lambda \frac{dT}{dr} S$, $S = 2\pi r L$

$$\frac{\dot{Q}_1}{L} = \frac{\lambda (T_1 - T_2)\, 2\pi}{\ln(R_e/R_i)} \quad (1)$$

Conveccion: $\dot{Q}_2 = h_i S_i(\theta_i - T_i) \rightsquigarrow \frac{\dot{Q}_i}{L} = h_i\, 2\pi R_i\,(\theta_i - T_i) \quad (2)$

$\dot{Q}_3 = h_e S_e(T_e - \theta_e) \rightsquigarrow \frac{\dot{Q}_e}{L} = h_e\, 2\pi R_e\,(T_e - \theta_e) \quad (3)$

El calor no se pierde, por lo tanto: $\dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3 = \dot{Q}$

Sumando (1), (2) y (3)

$$(\theta_i - \theta_e) = \frac{\dot{Q}}{L}\left(\frac{\ln(R_e/R_i)}{2\pi\lambda_H} + \frac{1}{h_i 2\pi R_i} + \frac{1}{h_e 2\pi R_e}\right) \rightsquigarrow \boxed{\frac{\dot{Q}}{L} = 137{,}63\ \frac{W}{m}}$$

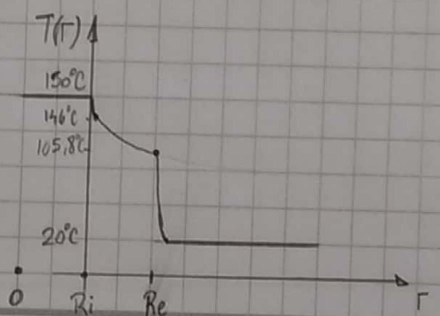
de (2) $\rightsquigarrow T_i = 146{,}23\ °C$

de (3) $\rightsquigarrow T_e = 105{,}89°C$

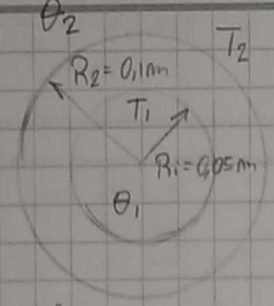
Perfil de Temperaturas

En (1), (2) y (3) se reemplaza T_2 por $T(r)$ y R_e por r

$$T(r) = \begin{cases} \theta_2 & 0 < r < R_i \\ -\frac{\dot{Q}}{L}\cdot\frac{\ln(r/R_i)}{\lambda \cdot 2\pi} + T_i & R_i < r < R_e \\ \theta_1 & r > R_e \end{cases}$$



4)



Para $R_x = 0{,}07\ m \rightsquigarrow \nabla T = -100\ \frac{°C}{m}$

$$\delta\dot{Q} = -\lambda \bar{\nabla} T \cdot d\bar{s} = -\lambda \nabla T \check{r}\, ds\, \check{r} = -\lambda \nabla T\, ds$$

a) $\frac{\dot{Q}}{L} = -\lambda \nabla T \cdot 2\pi R_x = 2813{,}55\ \frac{W}{m}$, $T_1 > T_2$

b) $\delta\dot{Q} = -\lambda \bar{\nabla} T \cdot d\bar{s} = -\lambda \frac{dT}{dr} \check{r}\, ds\, \check{r} = -\lambda \frac{dT}{dr} ds$

$$\delta\dot{Q} = -\lambda \frac{dT}{dr} ds \rightsquigarrow \dot{Q} = -\lambda \frac{dT}{dr} 2\pi r L$$

$$\frac{\dot{Q}}{L} \ln\left(\frac{R_2}{R_1}\right) = \lambda 2\pi (T_1 - T_2) \quad , \quad T_2 = 0°C$$

$$\frac{\dot{Q}}{L} \frac{\ln(R_2/R_1)}{\lambda 2\pi} = T_1 \longrightarrow T_1 = 4{,}852\ °C$$

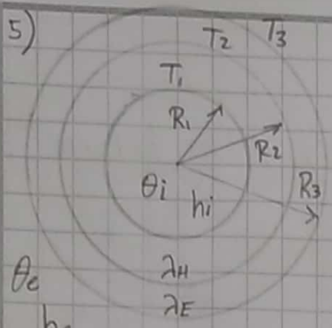
c) $$T(r) = \begin{cases} T_1 - \frac{\dot{Q}}{L} \frac{\ln(r/R_1)}{2\pi \cdot \lambda} & R_1 < r < R_2 \\ \theta_1 & 0 < r < R_1 \\ \theta_2 & r > R_2 \end{cases}$$

$$\frac{\dot{Q}}{L} = h_1 2\pi R_1 (\theta_1 - T_1) \rightsquigarrow \theta_1 = 389{,}22\ °C$$

$$\frac{\dot{Q}}{L} = h_2 2\pi R_2 (T_2 - \theta_2) \rightsquigarrow \theta_2 = -192{,}18°C \,.$$

Radio tal que $T = \frac{T_1 + T_2}{2} = 2{,}426°C \rightsquigarrow R = 0{,}071\ m$

5)



θ_e

h_e

$R_1 = 0{,}100\ m$ $\qquad \theta_i = 200°C$

$R_2 = 0{,}105\ m$ $\qquad T_3 = 50°C$

$R_3 = 0{,}155\ m$

Transmision:

$$\delta \dot{Q} = -\lambda \bar{\nabla} T \cdot d\bar{S} = -\lambda \nabla T\, \check{r}\, ds\, \check{r} = -\lambda \nabla T\, ds = -\lambda \frac{dT}{dr}\, ds$$

$$\dot{Q} = -\lambda \frac{dT}{dr} 4\pi r^2 \leadsto \frac{-\dot{Q}}{4\pi(R_2 - R_1)} = -\lambda (T_2 - T_1) \leadsto \frac{-\dot{Q}}{4\pi\lambda_H}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = -T_2 + T_1 \quad (1)$$

$$\leadsto \frac{-\dot{Q}}{4\pi\lambda_E}\left(\frac{1}{R_3} - \frac{1}{R_2}\right) = -T_3 + T_2 \quad (2)$$

Conveccion:

$$\dot{Q} = h S (T_c - T_F) \quad \begin{cases} \dot{Q} = h_i 4\pi R_1^2 (\theta_i - T_1) & (3) \\ \dot{Q} = h_e 4\pi R_3^2 (T_3 - \theta_e) & (4) \end{cases}$$

$$\Sigma(i),\ i = 1,2,3 \leadsto \theta_i - T_3 = \dot{Q}\left[\frac{1}{h_i 4\pi R_1^2} - \frac{1}{4\pi\lambda_H}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) - \frac{1}{4\pi\lambda_E}\left(\frac{1}{R_3} - \frac{1}{R_2}\right)\right] \quad (5)$$

$$\boxed{\dot{Q} = 15{,}75\ W}$$

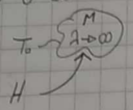
de (4) $\boxed{\theta_e = 41\ °C}$

de (2) $\boxed{T_2 = 164{,}3\ °C}$

Si $\dot{Q} = 14{,}175$

de (5), Busco R_3' $\leadsto R_3' = 0{,}165\ m$ $\qquad e_E = 0{,}06\ m$

6) $\theta = 20^{\circ}C$ $\quad M = 1\ Kg \quad S = 0{,}01\ m^2 \quad C_e = 840\ \frac{J}{Kg\,^{\circ}C}$ $\quad H = 58\ W\ , \quad T_0 = 40^{\circ}C \quad h = 10\ \frac{W}{m^2\,^{\circ}C}$



Al ser $\lambda \to \infty$, todo el cuerpo tiene la misma Temperatura $(\nabla T = 0)$ y por lo tanto cambiara su temperatura solo por conveccion.

Siendo $\quad Q = m\,C_e\,\Delta T$

$$\dot{Q} = m\,C_e\,\frac{dT}{dt}$$

y tambien $\quad \dot{Q} = H - h\,S\,(T_c - 20)$

Igualando,

$$m\,C_e\,\frac{dT(t)}{dt} = H - h\,S\,(T(t) - 20)$$

$$-\frac{m\,C_e}{hS}\,\frac{dT(t)}{dt} = -\frac{H}{hS} + T(t) - 20$$

$$\frac{dT(t)}{T(t) - \frac{H}{hS} - 20} = -\frac{hS}{mC_e}\,dt$$

$$\ln\left(T(t) - \frac{H}{hS} - 20\right) = -\frac{hS}{mC_e}\,t + K' \quad , \quad K' = \ln K$$

$$\ln\left(\frac{T(t) - H/hS - 20}{K}\right) = -\frac{hS}{mC_e}\,t$$

$$T(t) - H/hS - 20 = K\,e^{-\frac{hS}{mC_e}.t}$$

Siendo $T(t=0) = T_0 = 40$

$$T_0 - H/hS - 20 = K\,e^{0} \quad \rightsquigarrow \quad K = -560^{\circ}C$$

Por lo tanto $\quad T(t) = \frac{H}{hS} + 20 + K\,e^{-\frac{hS}{mC_e}.t}$

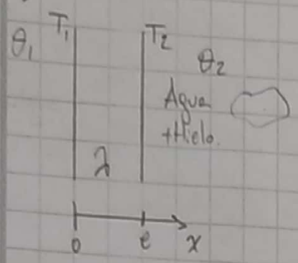
$$T(t) = 600^{\circ}C - 560^{\circ}C\,.\,e^{-t/8400}$$

Si $t \to \infty \rightsquigarrow T_f = 600^{\circ}C$

Si $T_f = 300^{\circ}C \rightsquigarrow t = 5243$ seg $\qquad$ Si $\tau = 8400 \rightsquigarrow t = 0{,}624\ \tau$

húsares

7) $S_T = 0{,}94\ m^2$



Datos:

$\theta_1 = 30°C$

$\lambda = 0{,}03\ W/m\,°C$

$h_1 = 5\ W/m^2\,°C$

$e = 0{,}03\ m$

$m_H = 40\ Kg$

$m_A = 20\ Kg$

$h_2 = 581{,}5\ \frac{W}{m^2\,°C}$

$C_F = 332{,}85 \cdot 10^3\ \frac{J}{Kg}$

Se asume que $\theta_2 = 0°C$ y despreciando la conveccion entre el agua y la pared

$\dot{Q} = \lambda S (T_1 - T_2)$ (1)

$\dot{Q} = h_1 S (\theta_1 - T_1)$ (2) $\dot{Q} = h_2 S (T_2 - \theta_2)$ (3)

$Q = \Delta m_H C_F \rightsquigarrow \dot{Q} = \frac{dm_H}{dt} C_F$ (4)

$$\theta_1 - \theta_2 = \dot{Q}\left(\frac{1}{\lambda S} + \frac{1}{h_1 S} + \frac{1}{h_2 S}\right)$$

$$\frac{\theta_1 - \theta_2}{\left(\frac{1}{\lambda S} + \frac{1}{h_1 S} + \frac{1}{h_2 S}\right)} = \frac{dm_H}{dt} C_F \rightsquigarrow \frac{(\theta_1 - \theta_2)\, t}{\left(\frac{1}{\lambda S} + \frac{1}{h_1 S} + \frac{1}{h_2 S}\right) \cdot C_F} = \Delta m(t)$$

Si $\Delta m = 1\ Kg \rightsquigarrow t = 395{,}82\ seg$

8) Cuerpo negro: $T_C = 33°C$, $S_C = 1,4\ m^2$

Habitación $T_A = 20°C$

Ley de Stefan-Boltzman

$$\dot{Q} = e\sigma . S\left(T_C^4 - T_A^4\right) \quad , \quad e = 1$$

Las Temp en Kelvin, luego

$$\boxed{\dot{Q} = 111\ W}$$

9) Cuerpo negro: $S_T = 1,22\ 10^{-3}\ m^2$

$I = 20\ A$

$V = 33,9\ V$

$$P_{OT} = I V = 678\ W = \dot{Q}$$

Asumiendo cero la temperatura del ambiente,

$$\dot{Q} = \sigma S\ T_F^4 \rightsquigarrow \boxed{T_F = 1767\ °K}$$

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1 litro atm = 101,3 J

3) Helio: H_2 → Gas ideal diatomico   Evolucion a $P = cte = 1\,atm$

1 mol H_2 — 8 g H_2 → $m = 1\,mol$

Estado inicial: $T_1 = 300°K$ $V_1 = V = 0{,}0246\ m^3$

" Final: $T_2 = 600°K$ $V_2 = 2V = 0{,}0492\ m^3$

$P_1 V_1 = m R T_1$ → $V_1 = 0{,}0246\ m^3$

$P_2 V_2 = m R T_2$ → $T_2 = 600°K$

$\frac{N}{m^2}\, m^3 = mol\, \frac{Joul}{mol\,K}\, K$

$$W = \int_{V_1}^{V_2} P\, dV = P(V_2 - V_1) = 2492\ J$$

$\Delta U = C_v\, m (T_2 - T_1)$, $C_v = 5/2 . R$ → $\Delta U = 6235{,}8\ J$

Como es a $P = cte$, vale que

$Q = C_P\, m (T_2 - T_1)$, $C_P = 7/2\ R$ → $Q = 8730{,}2\ J$

Se debe verificar el primer principio de la termodinamica

$$\Delta U = Q - W = 8730{,}2\ J - 2492\ J = 6235{,}8\ J$$

4) $\underline{P_1}$ $\underline{V_1}$ $\underline{T_1}$ Estado inicial   Evolucion de m moles de gas monoatomico adiabatico

P_2 $\underline{2V_1}$ T_2 Estado final $C_P = 2{,}5\,R$ $C_v = 1{,}5\,R$ $\gamma = \frac{C_P}{C_v} = 5/3$

$P_1 V_1^{5/3} = P\, V^{5/3}$

↳ $P = \frac{P_1 V_1^{5/3}}{V^{5/3}}$ $P_2 = \frac{P_1}{2^{1,67}}$

$\Delta U = m C_v (T_2 - T_1)$

$Q = 0$

$$W = \int_{V_1}^{V_2} P\, dV = P_1 V_1^{5/3} \int_{V_1}^{2V_1} \frac{1}{V^{5/3}}\, dV = -P_1 V_1^{5/3}\, 3/2 . \left[\frac{1}{(2V_1)^{2/3}} - \frac{1}{V_1^{2/3}} \right] = -\frac{P_1 V_1^{5/3}}{V_1^{2/3}}\, 3/2 \left(\frac{1}{2^{2/3}} - 1 \right)$$

$$W = \frac{3 P_1 V_1}{2} \left(1 - \frac{1}{2^{2/3}} \right) = 0{,}557\ P_1 V_1 = 0{,}557\ m R T_1$$

$$T_2 = \frac{\frac{P_1}{2^{1,67}}\, 2 V_1}{m R} = \frac{P_1 V_1}{2^{0,67}\, m R} = \frac{\cancel{m} R T_1}{2^{0,67}\, \cancel{m R}} = \frac{T_1}{2^{0,67}}$$

$$\Delta U = -0{,}557\ P_1 V_1 = -0{,}557\ m R T_1$$

$$\Delta U = m\, 1{,}5 R \left(\frac{T_1}{2^{0,67}} - T_1 \right) = m R T_1\, 1{,}5\, (-0{,}37) = -0{,}557\ m R T_1$$

5) Gas ideal, $m = 1$ mol

A: $P_A =$, $V_A = V$, $T_A = T$

B: $P_B = P$, $V_B = 2V$, $T_B = T$

C: $P_C = P$, $V_C = V$, $T_C = T/2$

D: $P_D =$, $V_D = V$, $T_D = T$

$$W_{AB} = \int_V^{2V} \frac{mRT}{V}\, dV = mRT \ln(2)$$

$$P = \frac{mRT}{2V} = \frac{mRT'}{V} \Rightarrow T' = \frac{V}{2V} T = \frac{T}{2}$$

$$W_{BC} = P(V - 2V) = -PV = -\frac{mRT}{2}$$

$$W_{CD} = 0$$

$$W_{DA} = 0$$

$$W_T = mRT\left(\ln(2) - 1/2\right)$$

6) 1-2 Isoterma

2-3 Isocora

3-1 Adiabatica

	1	2	3
P (MPa)	0,71	0,1065	0,0499
V (m^3)	0,0015	0,01	0,01
T (K)	300	300	140

$$P_1 V_1^{1,4} = P_3 V_3^{1,4} \rightsquigarrow P_3 = 0{,}0499 \text{ MPa}$$

$$m = 11{,}95 \text{ g} \cdot \frac{1 \text{ mol}}{28 \text{ g}} = 0{,}427 \text{ mol}$$

$N_2 \rightsquigarrow C_P = 3{,}5R$

$\quad \searrow C_V = 2{,}5R$

$$\left.\begin{aligned} \Delta U_{12} &= 0 \\ \Delta U_{23} &= m C_V (T_3 - T_2) = -1420{,}1 \text{ J} \\ \Delta U_{31} &= m C_V (T_1 - T_3) = 1420{,}1 \text{ J} \end{aligned}\right\} \boxed{\Delta U_T = 0}$$

$$\left.\begin{aligned} Q_{12} &= W_{12} = \int_{V_1}^{V_2} \frac{mRT}{V}\, dV = mRT \ln(V_2/V_1) = 2020{,}6 \text{ J} \\ Q_{23} &= m C_V (T_3 - T_2) = -1420{,}1 \text{ J} \\ Q_{31} &= 0 \end{aligned}\right\} Q_T = 600{,}5 \text{ J}$$

$$-\frac{P_1 V_1^{1,4}}{0{,}4} V^{-0,4} \qquad 11?$$

$$\left.\begin{aligned} W_{12} &= 2020{,}6 \text{ J} \\ W_{23} &= 0 \\ W_{31} &= \int_{V_3}^{V_1} \frac{P_1 V_1^{1,4}}{V^{1,4}}\, dV = -\frac{P_1 V_1^{1,4}}{0{,}4} \cdot \left(\frac{1}{V_1^{0,4}} - \frac{1}{V_3^{0,4}}\right) = -1415{,}9 \text{ J} \end{aligned}\right\} W_T = 604{,}7 \text{ J}$$

7) Expansion de gas ideal monoatomico: $V_a \to 2V_a$, m, T_a ⇝ Datos

a) $P =$ cte

$$\boxed{W = \int_{V_a}^{2V_a} P\, dV = P(2V_a - V_a) = P.V_a = mRT_a}$$

$$P = \frac{\cancel{m}RT_a}{\cancel{V_a}} = \frac{\cancel{m}RT_b}{2V_a}$$

$Q = m\, C_P (T_b - T_a)$, $T_b = 2T_a$, $C_P = 2{,}5\, R$

$$\boxed{Q = m\, C_P\, T_a}$$

b) $T =$ cte

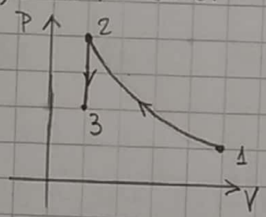
$$\Delta U = 0 \leadsto Q = W = \int_{V_a}^{2V_a} \frac{mRT_a}{V}\, dV = mRT_a \ln(2)$$

c) $Q = 0$

$$W = -\Delta U = \int_{V_a}^{2V_a} \frac{P_a V_a^{1{,}67}}{V^{1{,}67}}\, dV \quad , \quad P_b V_b^{1{,}67} = P_a V_a^{1{,}67}$$

$$= -\frac{P_a V_a^{1{,}67}}{0{,}67}\left(\frac{1}{V_b^{0{,}67}} - \frac{1}{V_a^{0{,}67}}\right) = -\frac{P_a V_a}{0{,}67}(-0{,}371) = 0{,}554\, P_a V_a = 0{,}554\, mRT_a$$

8) Helio. $m = 1$ mol



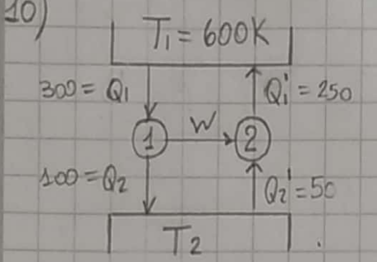
	1	2	3
P(MPa)	0,1	0,51	0,31
V(m³)	0,023	0,0072	0,0072
T(K)	273	441,6	273

$$\boxed{P_F = 0{,}31\ MPa}$$

9)

Guía 13

10)



$Q_1 = W + Q_2$ $\qquad$ $Q_1' = Q_2' + W$

$W = 200 \, J$ $\qquad$ $Q_1' = 250 \, J$

Desigualdad de Clausius para la maquina 1:

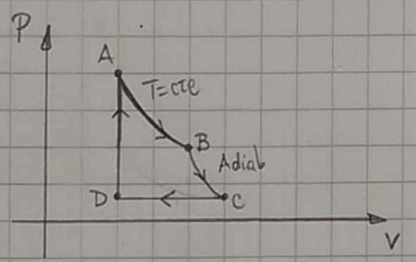
a) $\sum \frac{Q}{T} = 0$ dado que (1) es reversible $\rightsquigarrow \frac{300}{600} - \frac{100}{T_2} = 0 \rightsquigarrow T_2 = 200 \, K$

b) $\eta_1 = \frac{W}{Q_1} = 0{,}67$, $\varepsilon_2 = \frac{Q_2'}{W} = 0{,}25$

c) Desigualdad de Clausius para la maquina 2:

$\sum \frac{Q}{T} = \frac{50}{200} - \frac{250}{600} = -0{,}167$ no es reversible, es posible

11)



$P_C = P_D \rightsquigarrow \frac{mRT_C}{V_C} = \frac{mRT_D}{V_D} \rightsquigarrow V_C = \frac{T_C}{T_D} V_D =$

$V_A = V_D$ $\qquad$ $\frac{mRT_A}{P_A} = \frac{mRT_D}{P_D} \rightsquigarrow T_D = \frac{T_A P_D}{P_A}$

$\frac{1}{2} \frac{T_A}{T_A} \frac{V_A}{P_D} \cdot P_A = V_C$

AB: $Q_{AB} = W_{AB} = mRT_A \ln(2) = 2274 \, J$, $\Delta U = 0$

BC: $Q_{BC} = 0 \rightsquigarrow \Delta U_{BC} = m \, C_V (T_C - T_B) = m \, 1{,}5 R \, T_A . (-0{,}5) = -mRT_A \; 0{,}75 = -1845 \, J$

$W_{BC} = -0{,}75 \, mRT_A = -1845 \, J$

CD: $W = P_C (V_D - V_C) = -1314 \, J$

$\Delta U = m C_V (T_D - T_C) = -2020 \, J$

$Q = m C_p (T_D - T_C) = -3367 \, J$

	A	B	C	D
P(MPa)	1,64	0,82	0,146	0,146
V(m³)	0,002	0,004	0,011	0,002
T(K)	394,5	394,5	197,25	35

$0{,}82 \; 0{,}004^{1{,}67} = P_C \left(\frac{mRT_C}{P_C} \right)^{1{,}67}$

DA: $W = 0$

$\Delta U = m C_V (T_A - T_D) = 4484 \, J$

$Q = \Delta U = 4484 \, J$

$\Delta U_T = 0$ ✓

$W_T = Q_T$ ✓

Es una maquina motora: $\eta = \frac{W}{Q_{ent}} = \frac{W_T}{Q \oplus} =$

14) 1 mol de gas diatómico $C_p = 3,5R$ $C_v = 2,5R$

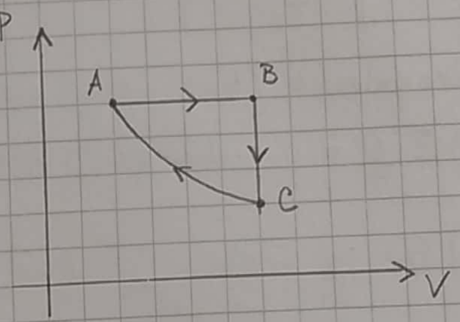
	A	B	C
P	P_A	P_A	$P_A(T_A/T_B)^{1,4}$
V	V_A	RT_B/P_A	RT_B/P_A
T	$\frac{P_A V_A}{R}$	T_B	$T_B(T_A/T_B)^{1,4}$

$$P_C\left(\frac{RT_B}{P_A}\right)^{1,4} = P_A \cdot V_A^{1,4}$$

$$P_C = P_A\left(\frac{V_A P_A}{RT_B}\right)^{1,4} = P_A\left(\frac{T_A}{T_B}\right)^{1,4}$$

$$T_C = \frac{1}{\cancel{n}R} \cdot \cancel{P_A}\left(\frac{T_A}{T_B}\right)^{1,4} \frac{T_B}{\cancel{P_A}} \cdot \cancel{R} = T_B\left(\frac{T_A}{T_B}\right)^{1,4}$$

AB: $P = cte$



- $\Delta U = C_v(T_B - T_A) = 2,5R\left(T_B - \frac{P_A V_A}{R}\right)$
- $W = P(V_B - V_A) = P_A\left(\frac{RT_B}{P_A} - V_A\right)$
- $Q = C_p(T_B - T_A) = 3,5R\left(T_B - \frac{P_A V_A}{R}\right)$

BC: $V = cte$

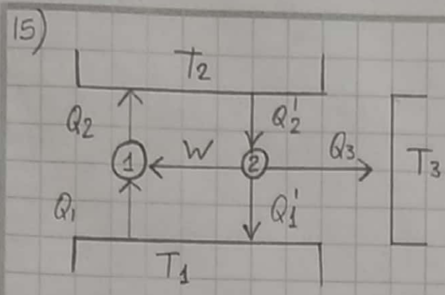
- $\Delta U = C_v(T_C - T_B) = 2,5R\left[T_B\left(\frac{T_A}{T_B}\right)^{1,4} - T_B\right]$
- $W = 0$
- $Q = -\Delta U$

CA: Adiab

- $\Delta U = C_v(T_A - T_C) = 2,5R\left[\frac{P_A V_A}{R} - T_B\left(\frac{T_A}{T_B}\right)^{1,4}\right]$
- $Q = 0$
- $W = -\Delta U$

c) Es una maquina motora dado que el trabajo neto es positivo

15)



$T_1 = 573\ K$

$Q_1 = 100\ J$ $\quad Q_1' = 50\ J$

$Q_2' = 200\ J$

M_1 reversible

$\varepsilon = 0{,}75$

a) (1) $Q_1 + W = Q_2$

(2) $\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$

(3) $\varepsilon = \frac{Q_1}{W}$

de (3) $W = 133{,}33\ J$

de (1) $Q_2 = 233{,}33\ J$

de (2) $T_2 = 1337\ K$

La maquina 1 es frigorífica

b) (4) $\frac{Q_2'}{T_2} - \frac{Q_1'}{T_1} - \frac{Q_3}{T_3} = 0$

(5) $Q_2' - Q_3 - Q_1' - W = 0$

de (5) $Q_3 = 16{,}667\ J$

de (4) $T_3 = 267{,}4\ K$